Introduction to nuclear structure:
an up-to-date perspective

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References:
"Fundamentals of Nuclear Models: Foundational Models"
--David J. Rowe and JLW, World Scientific, 2010 [Rowe & Wood—R&W]

"Shape Coexistence in Atomic Nuclei"
--Kris Heyde and JLW, Reviews of Modern Physics Vol. 83 1467 2011
[Heyde & Wood]

Challenges to the student* of nuclear structure

- Nucleus--quantum mechanical many-body problem

  not a branch of pedagogical quantum mechanics

  needs approximations or models (Pauli exclusion principle severely modifies bare nucleon-nucleon interaction)

- Nuclear structure study as a many-body problem: necessitates acquisition of large, systematic data sets

  we possess extraordinary control over the many-body aspects of the nucleus and the processes that can be effected in the laboratory

*We are all students

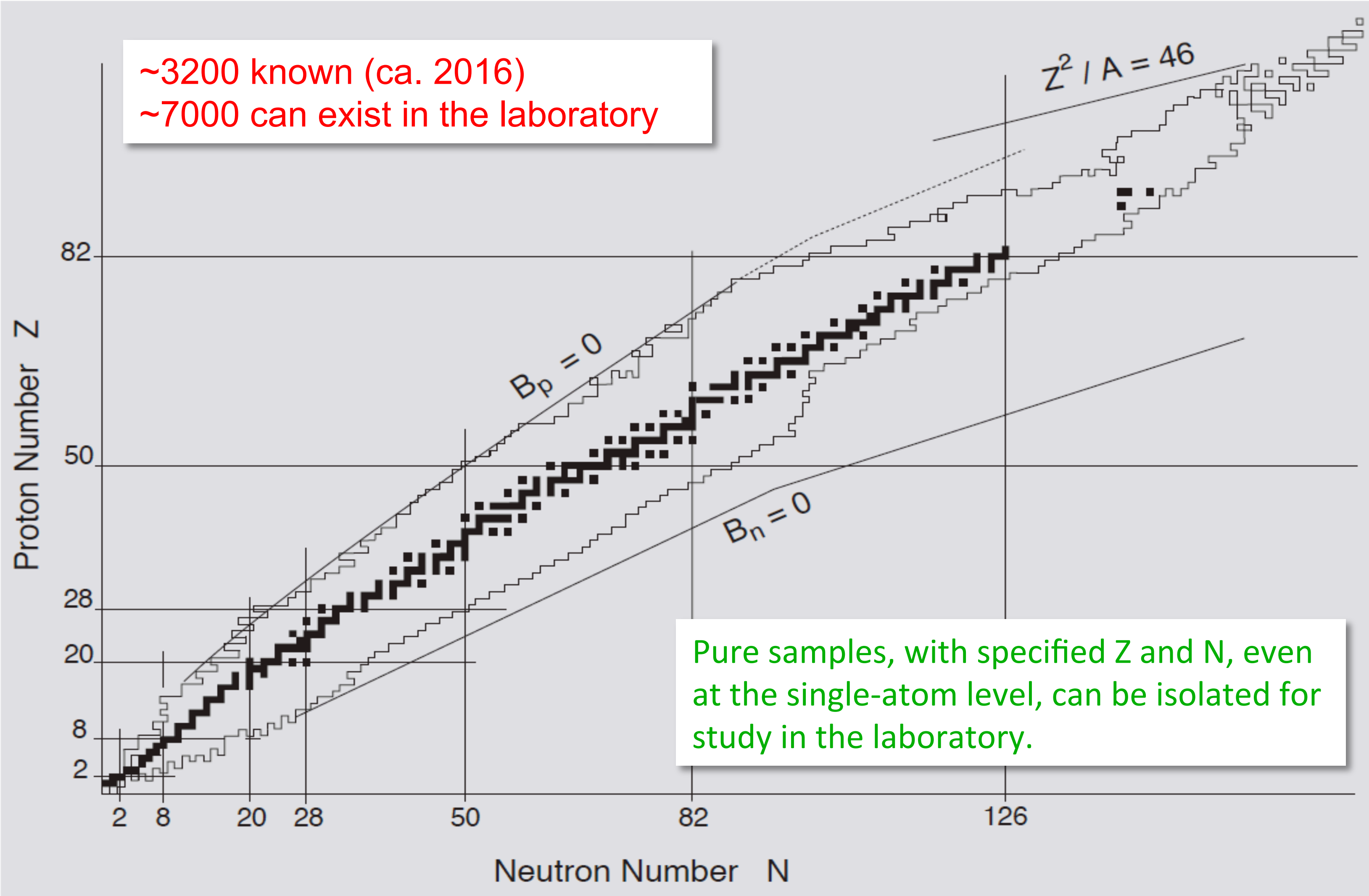
Philosophical perspective

- We are about to enter a restaurant that serves (brain) food.
- The instructor is the chef.
- But the chef does not cook and serve you; rather, the chef shows you *how you will cook your meal*.
- It is a (very) long time since the chef has been served a ready to eat meal.
- The chef has favorite recipes (they may not be yours).
- The chef still occasionally suffers from food poisoning (you will too).
- Sometimes what you cook will be delicious; sometimes it will be difficult to chew, and sometimes utterly indigestible.
- BON APPÉTIT!
- P.S. This is reality (television).

Acknowledgement: I would like to thank Marcus Scheck for challenging me to think more deeply about what I do when I present myself as a teacher.

The responsibility of a teacher is that their students do not (completely) starve.

Chart of the nuclides



R&W Fig. 1.2

Welcome to the many-body problem (restaurant)!

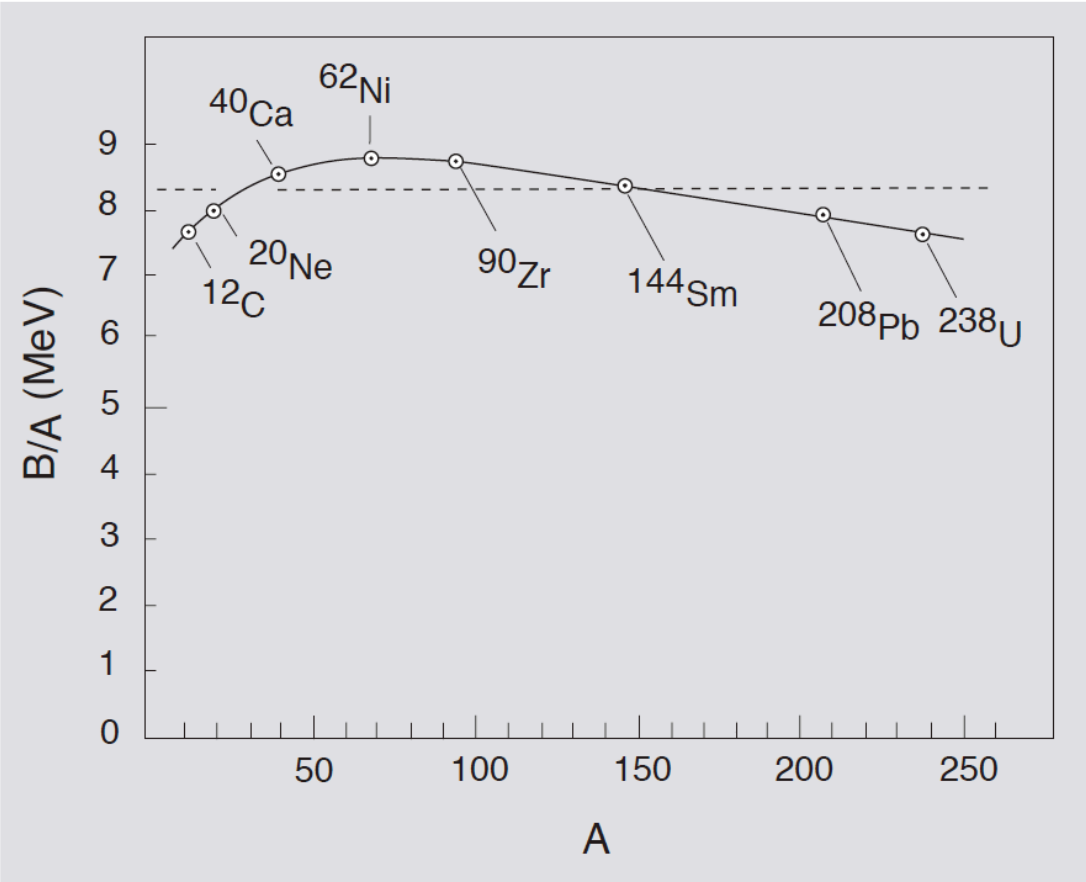
LECTURE 1

Nuclear systematics

Key types of data

Some basic features

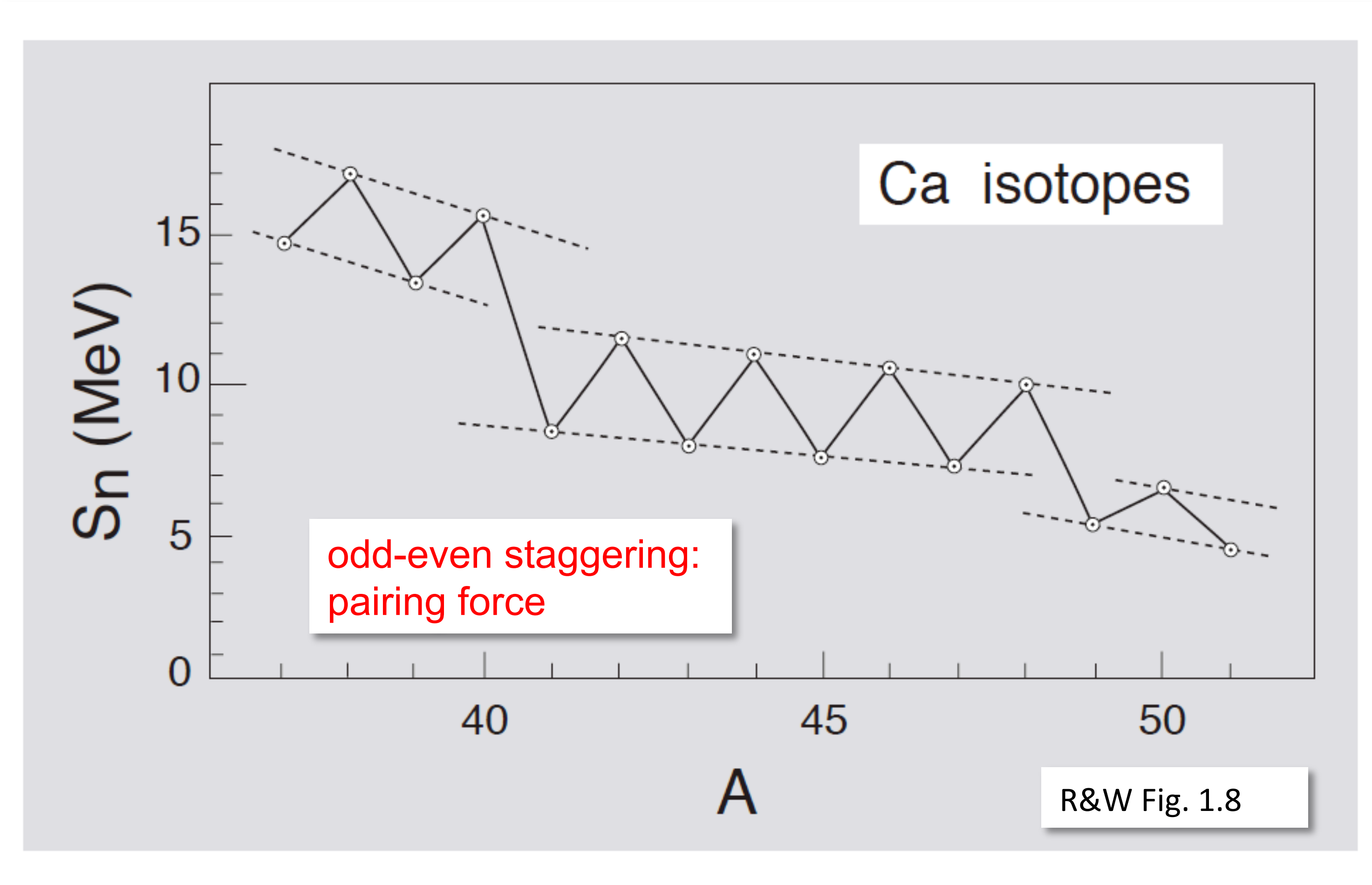
Nuclear binding energy per nucleon



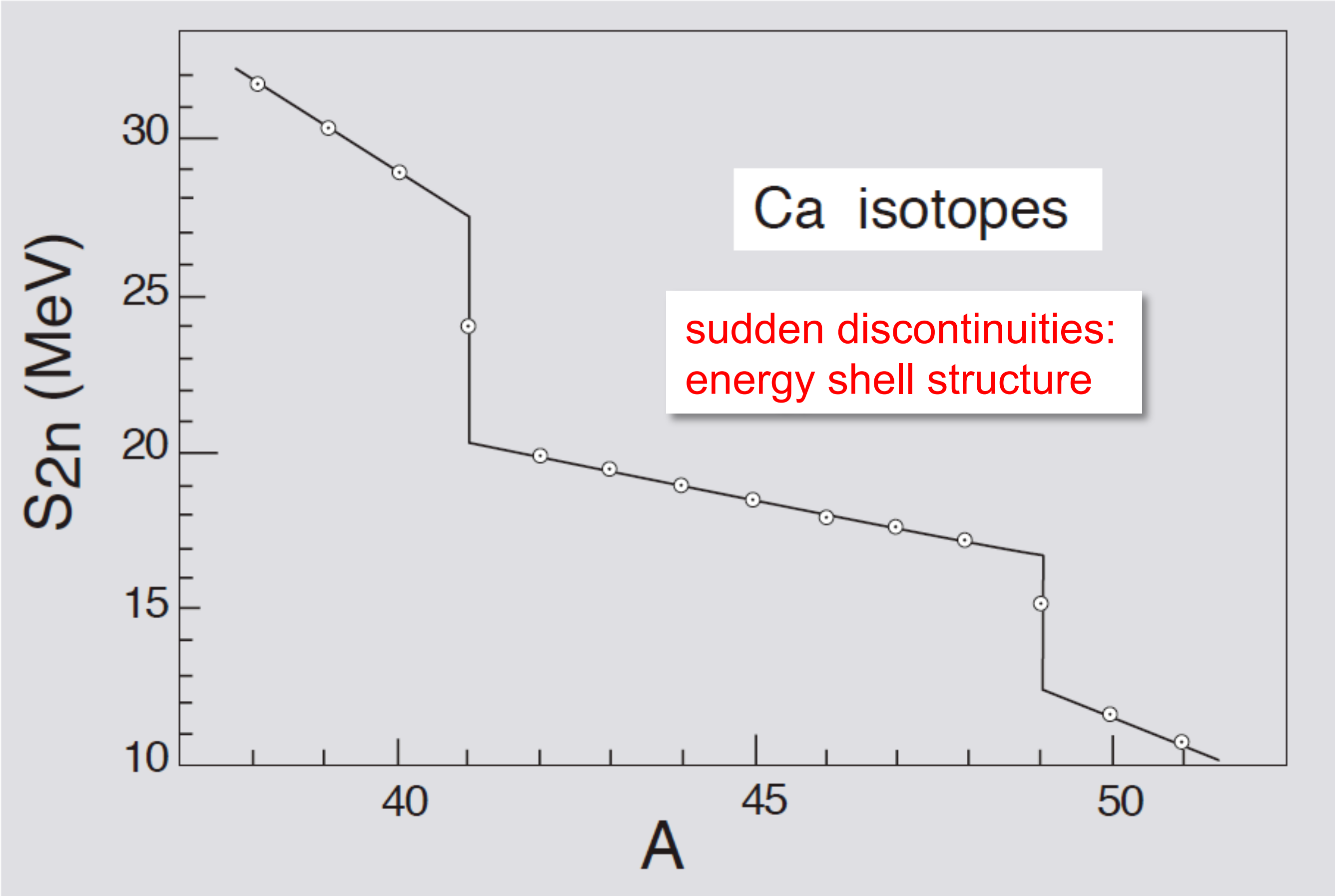
B/A--near constant value: nuclei are liquid-drop like

R&W Fig. 1.4

One-neutron separation energies

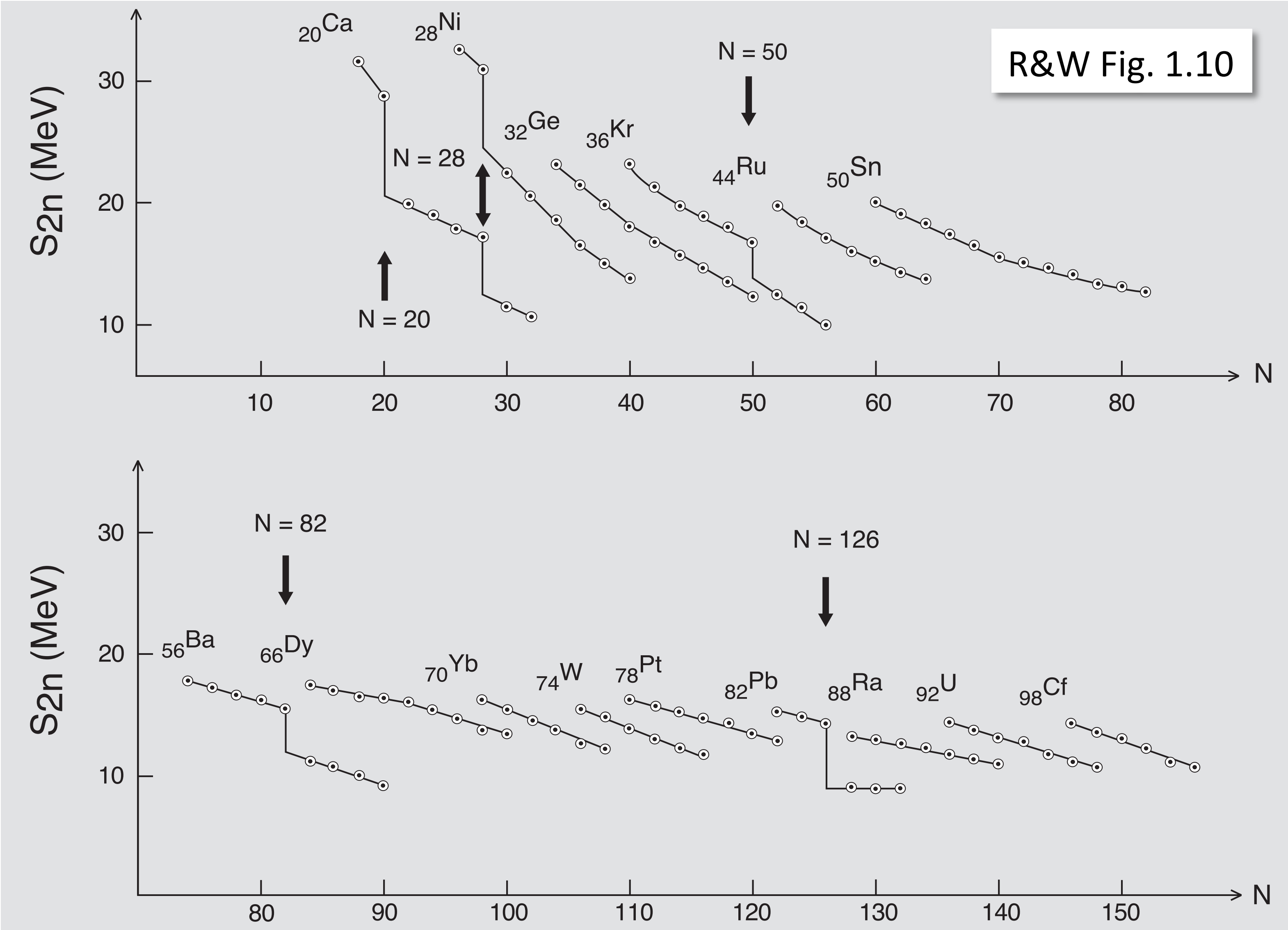


Two-neutron separation energies

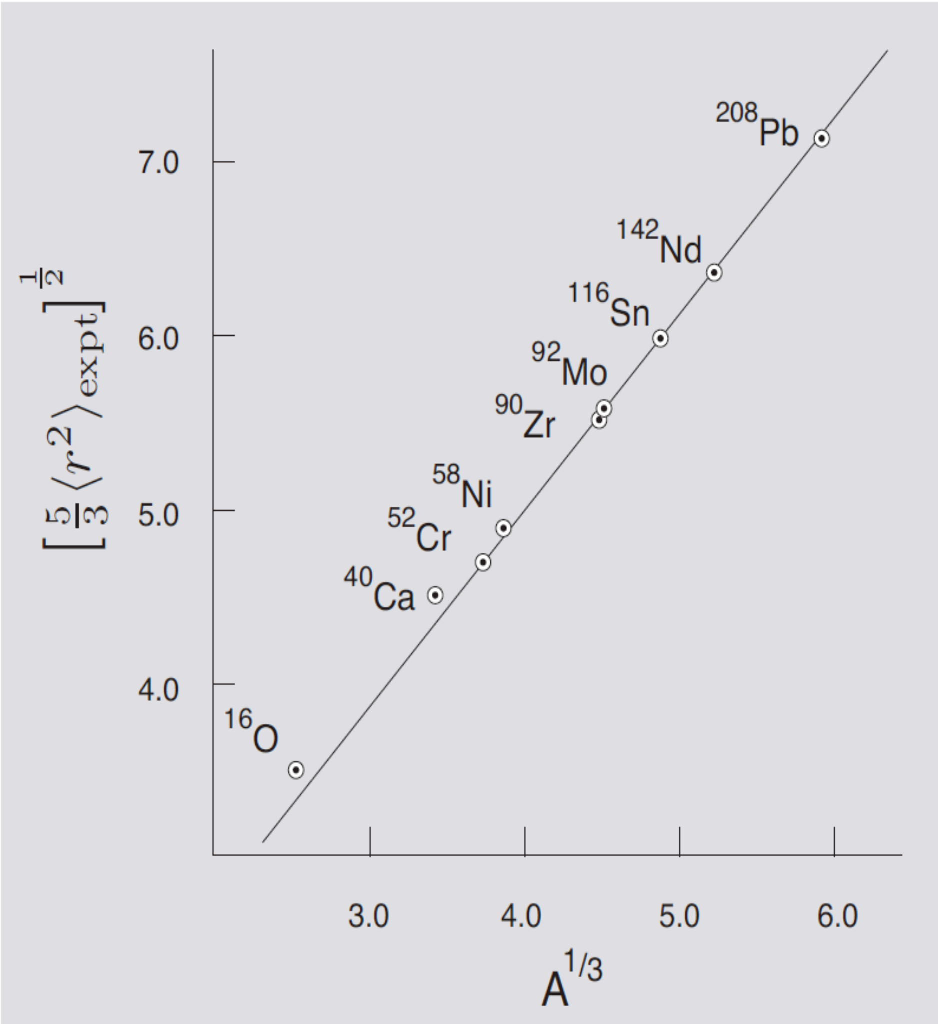


R&W Fig. 1.9

Differences in binding energy reveal shell structure



Root-mean-square charge radii of nuclei



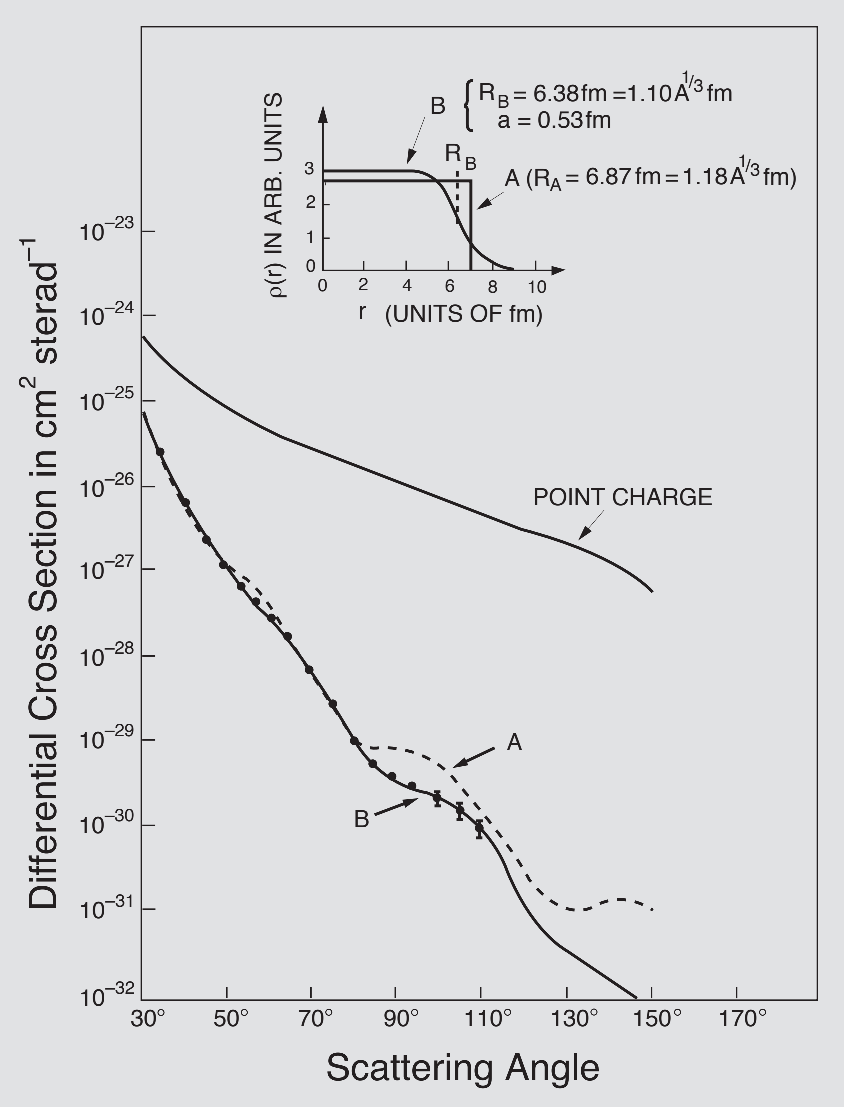
near linear slope:
nuclei are liquid-drop like

$$\boxed{R \approx 1.2\ A^{1/3}\ \text{fm}}$$

the size of the nucleus
fixes *its scale of quantization*

R&W Fig. 1.5

Elastic scattering of electrons: high precision probe of nuclear size (^{197}Au)



Elastic scattering of electrons reveals, via the fitting of model* charge-density distributions, that nuclei do not have sharp surfaces. E_e = 153 MeV.

$$^{*}\ \rho(r) = \rho_0 \left\{ 1 + \exp\left(\frac{r - R_B}{a} \right) \right\}^{-1}$$

λ(nm)= 1240/E(eV), for E >> $m_e c^2$ = 0.5 MeV

E_e = 153 MeV → λ = 8 fm

R&W Fig. 1.6

Nuclei do not have sharp surfaces: their surfaces are diffuse

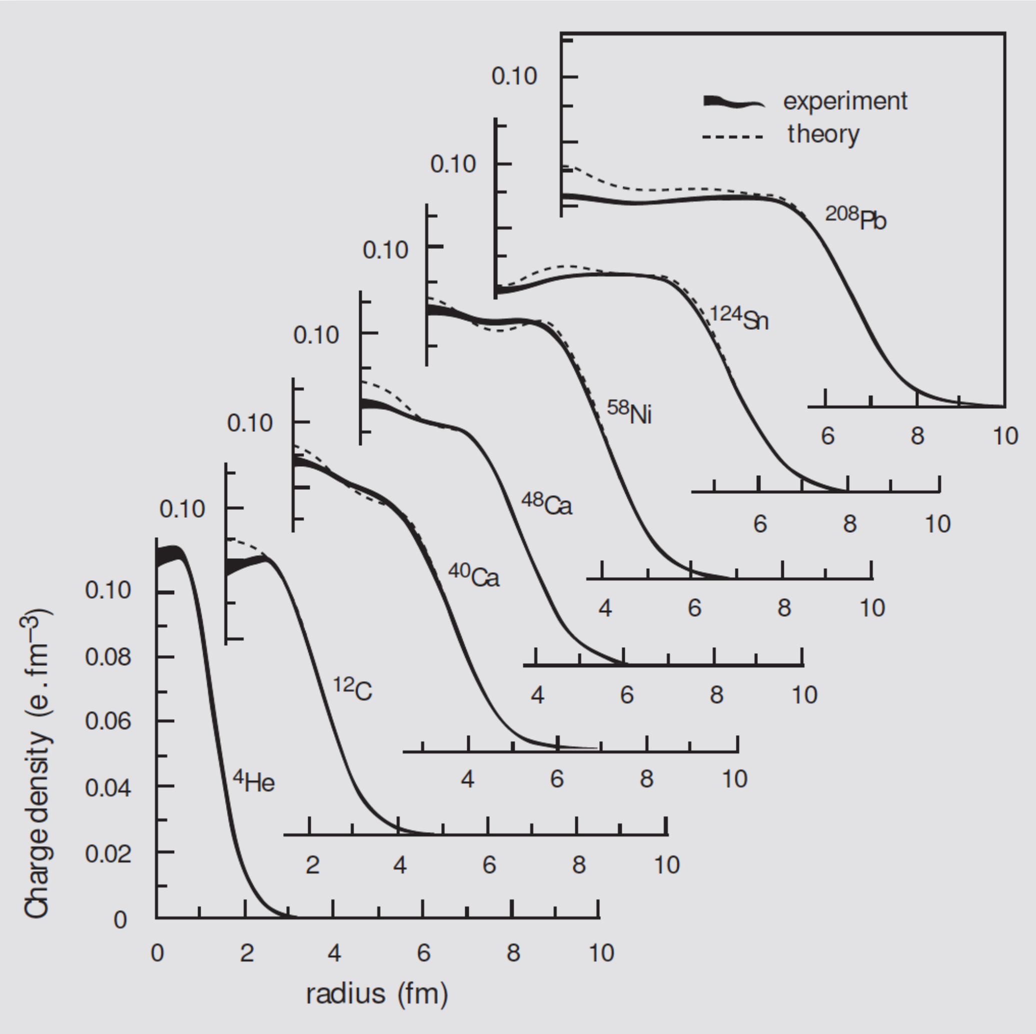
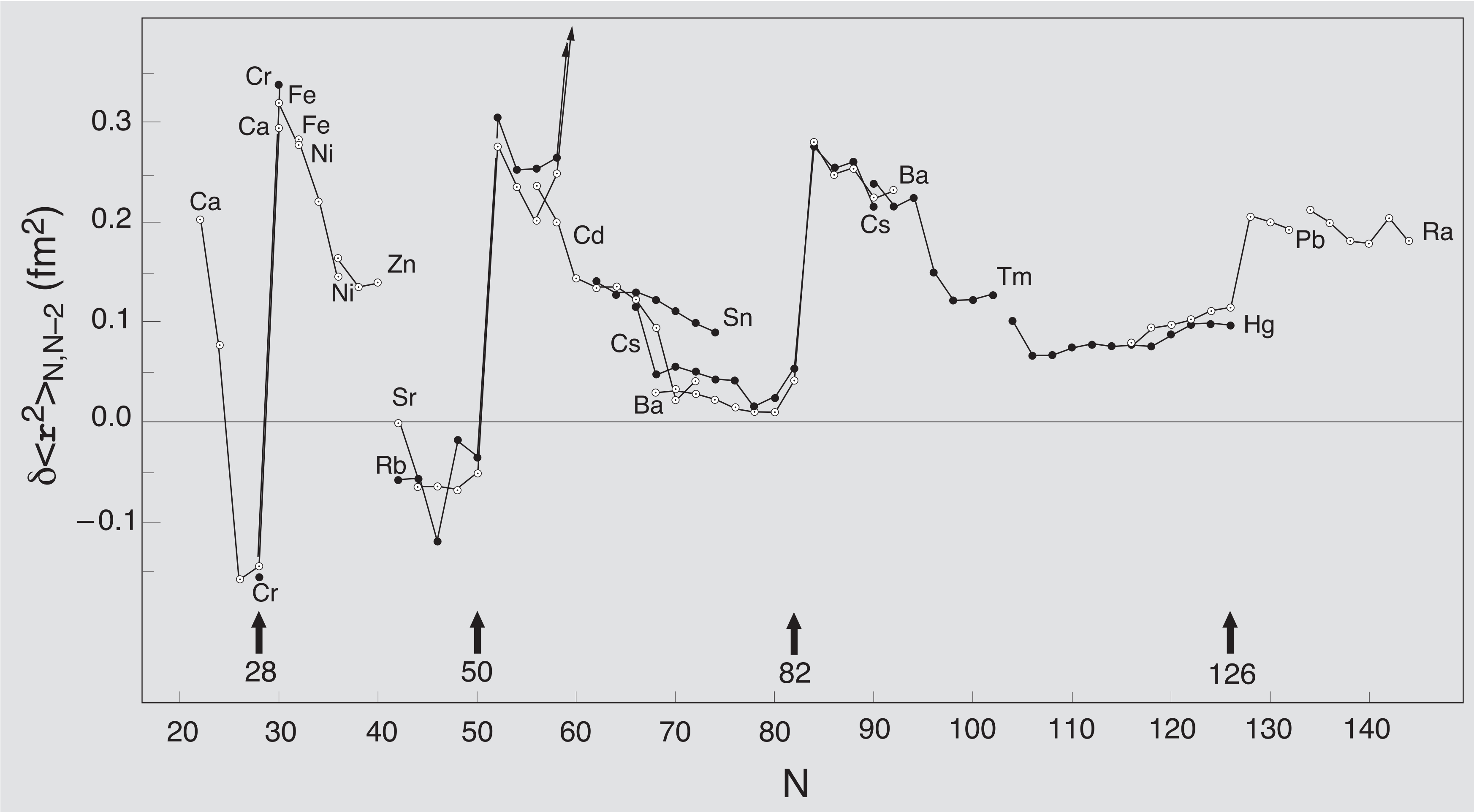


Figure: adaptation from B. Frois and C. Papanicolas, Ann. Rev. Nucl. Sci. **37** 133 1987

From elastic electron scattering

R&W Fig. 1.7

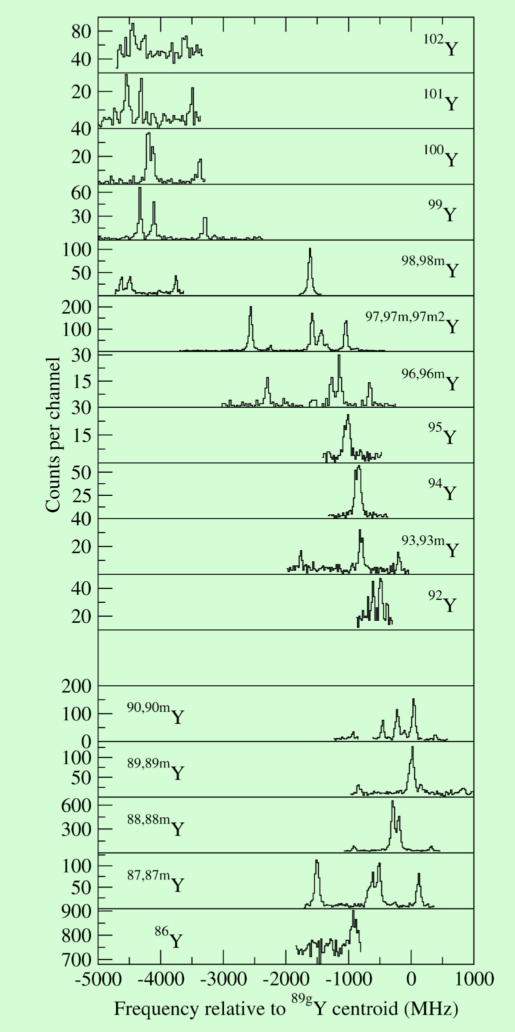
Differences in mean-square charge radii reveal shell structure

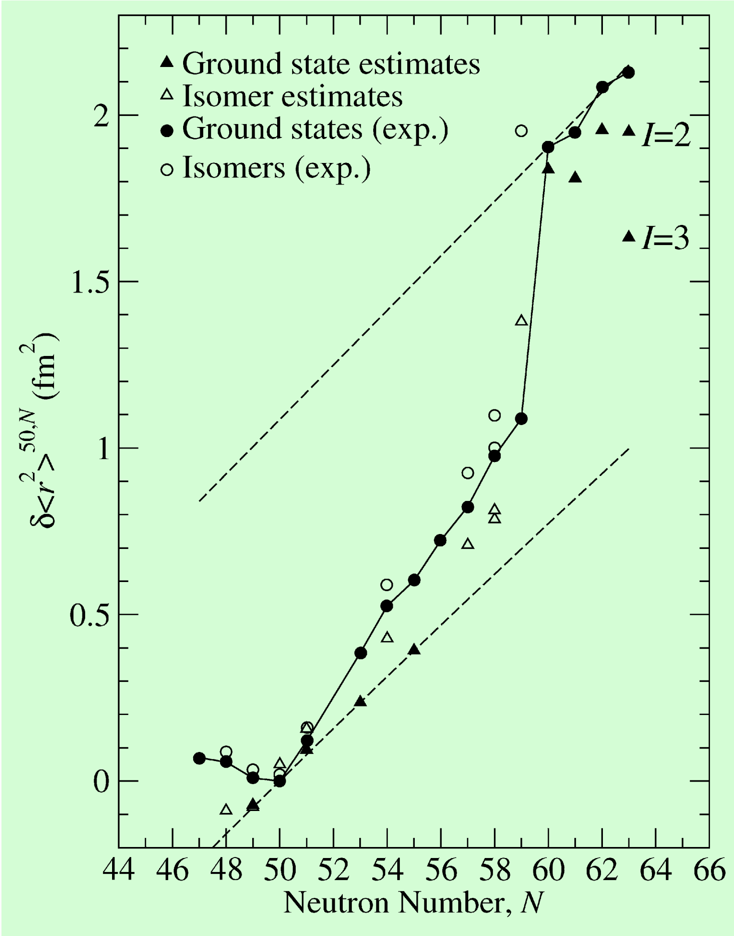


R&W Fig. 1.11

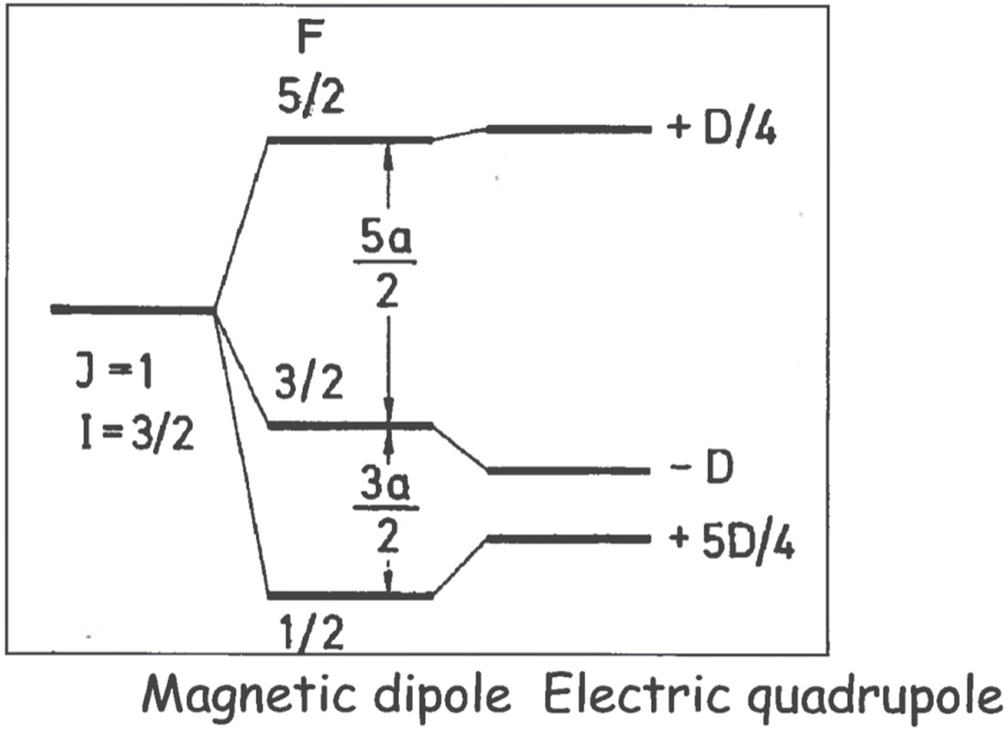
Atomic hyperfine spectroscopy leads to high-precision isotope shift data: Y (Z = 39) isotopes

B. Cheal et al., PL B645, 133 (2007)





Atomic hyperfine splitting due to magnetic dipole and electric quadrupole moments of the nucleus



$$a = g_I \mu_N B_J / \sqrt{J(J+1)}$$

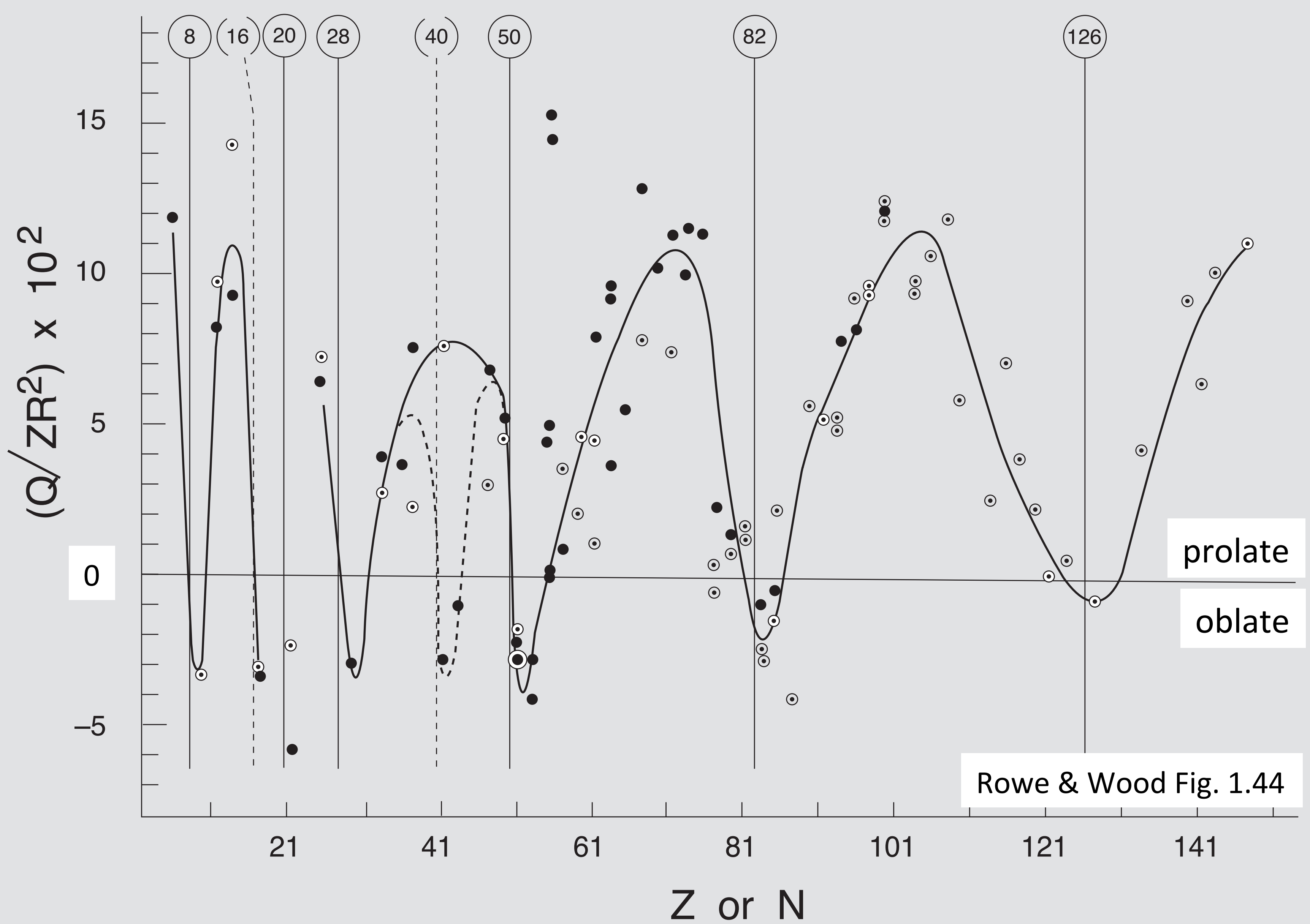
$$D = (eQ)(\partial^2 V/\partial z^2)$$

I – nuclear spin
J – electron spin
F – atomic spin

K. Heyde & JLW, Phys. Scr. **91** 083008 2016
J.A.S. Smith, Chem. Soc. Rev. **15** 225 1986

EARLY HISTORY:
Schuler and Schmidt 1935
Casimir 1935

Electric quadrupole moments indicate that many nuclei are non-spherical: odd Z *and* odd N



Multi-step Coulomb Excitation

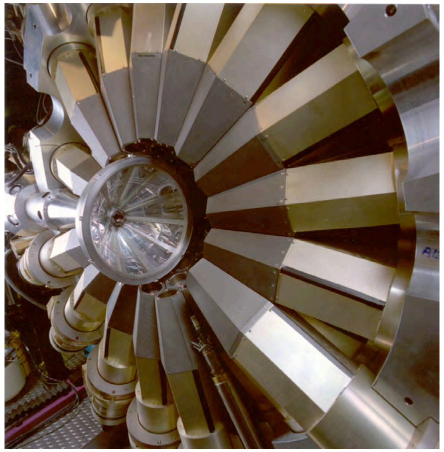
Multiple-step Coulomb excitation (multi-Coulex) populates excited collective states in nuclei.

An incident nucleus is scattered by the Coulomb interaction with a target nucleus.

A close ("safe") approach results in multiple Coulomb interactions only (no complication from strong interactions).

Level energies and transition strengths are deduced through γ-ray spectroscopy.

Level lifetimes, quadrupole moments and transition matrix elements are deduced by a least-squares fit to γ-ray yields.



Gammasphere:
110 Ge γ-ray detectors.

CHICO:
particle detector.

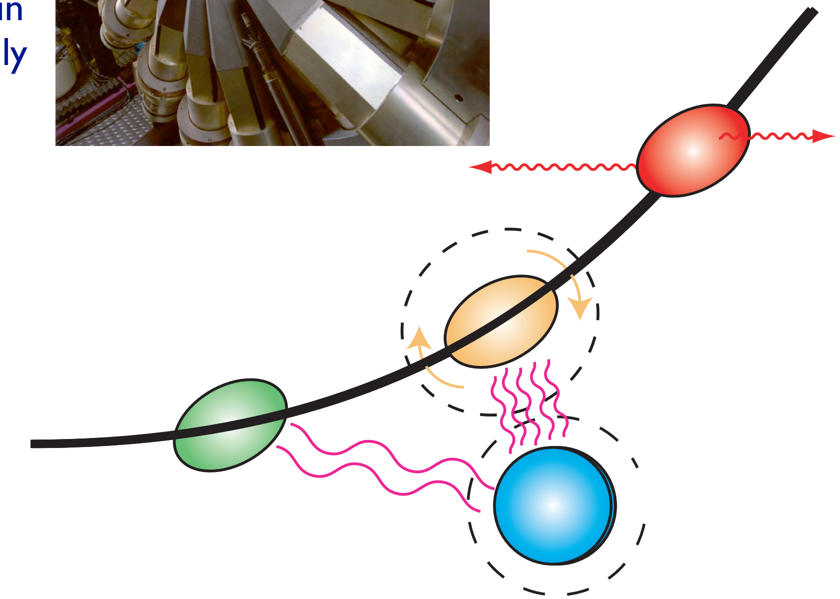
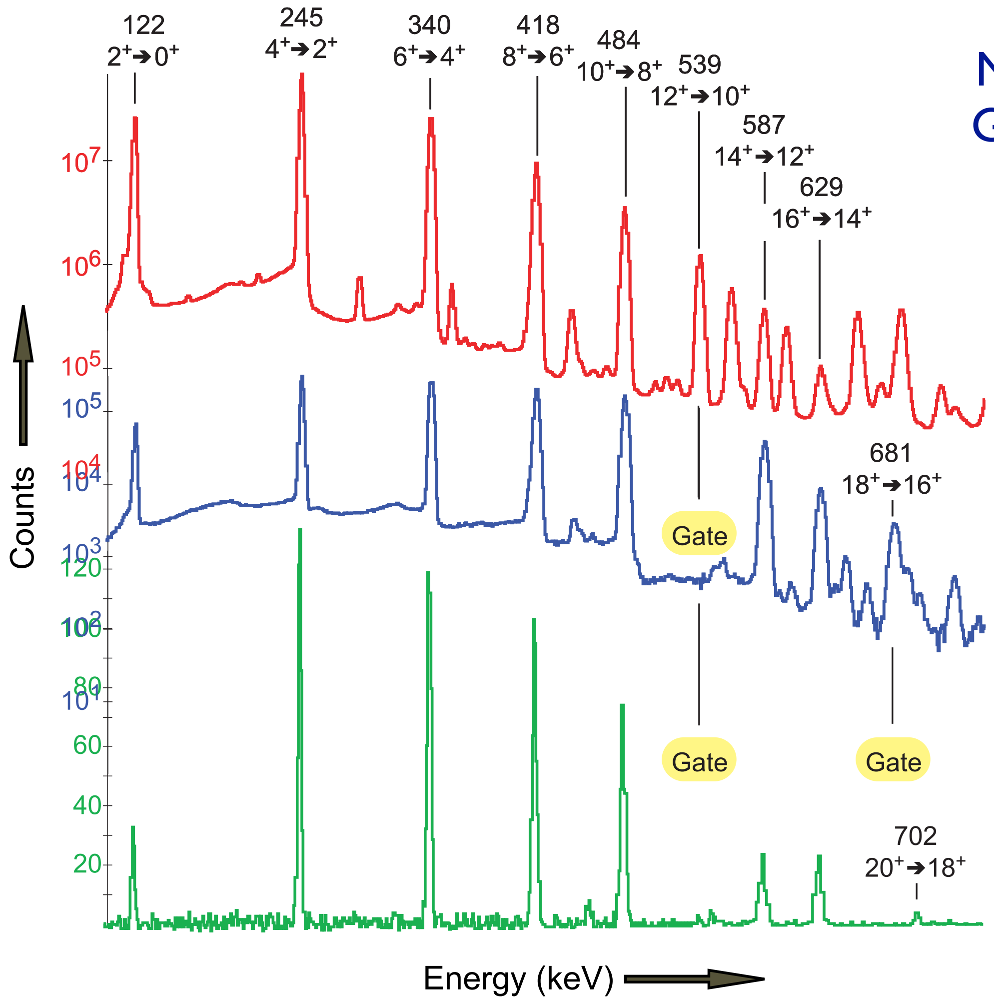


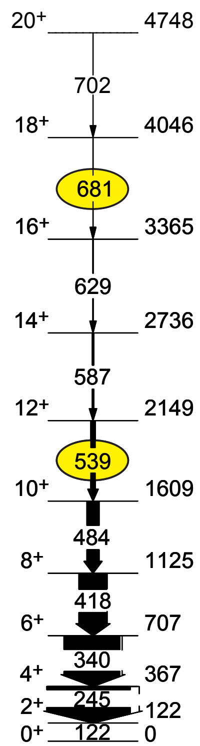
Figure: W.D. Kulp, Ga Tech

Multi-step Coulomb excitation: Doppler-corrected response of a beam of ^{152}Sm on a ^{208}Pb target

Figure: W.D. Kulp, Ga Tech



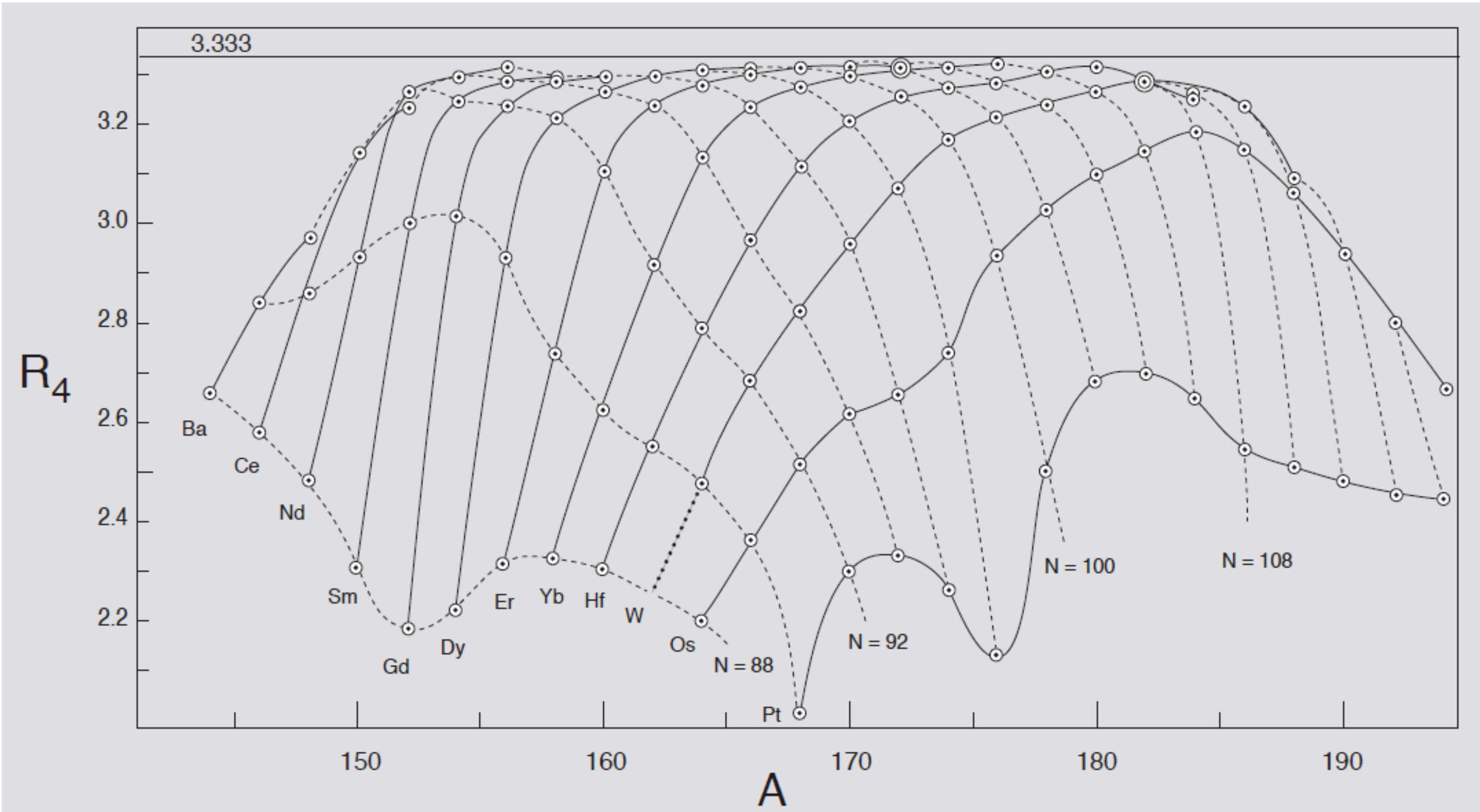
Multi-Coulex with Gammasphere and CHICO



Rigid rotor model energy ratios

$R_4 = E(4_1^+) / E(2_1^+)$ $\quad$ $E(I) = A\,I(I + 1)$

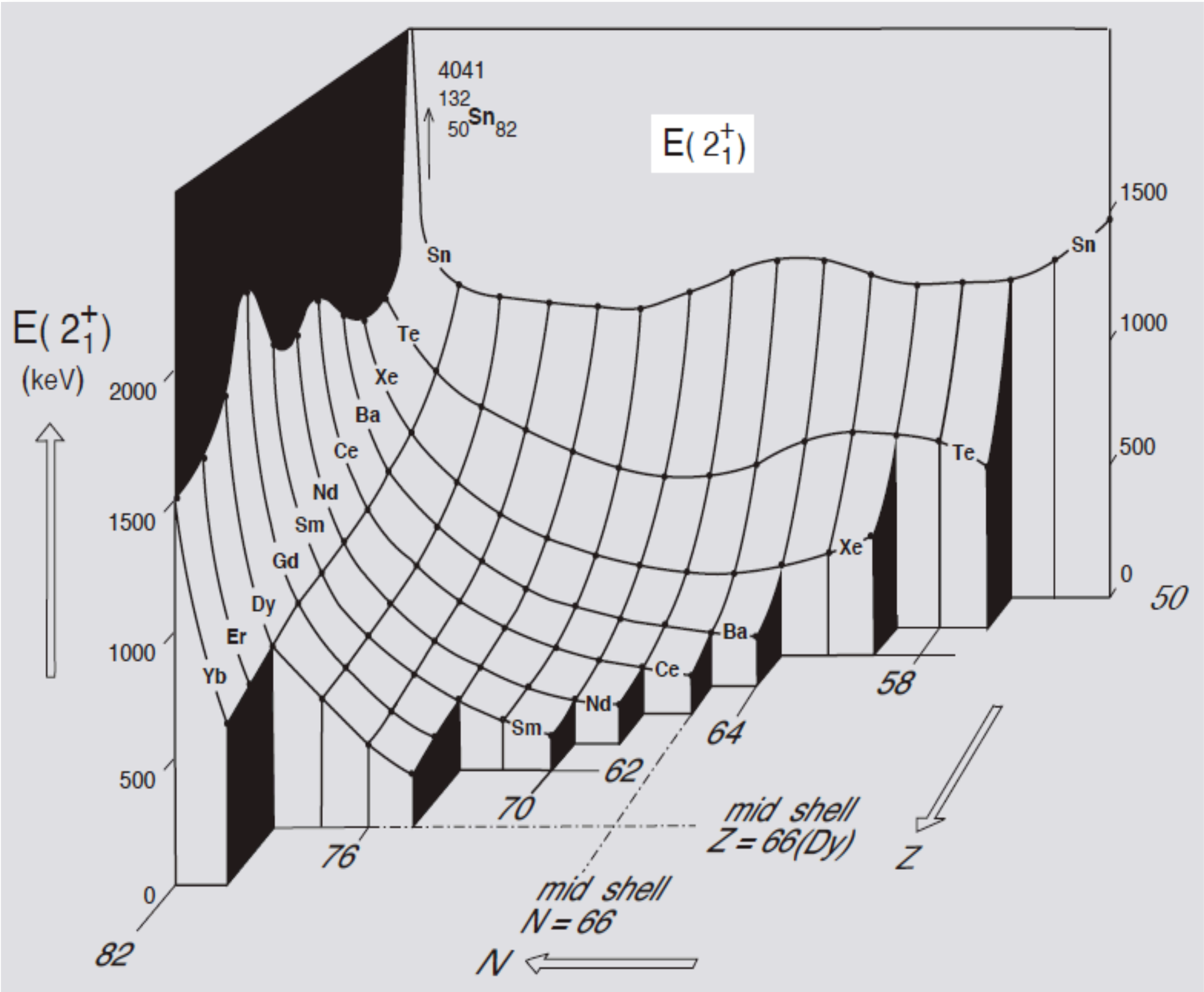
$A \sim (\text{moment of inertia})^{-1}$



R&W Fig. 1.56

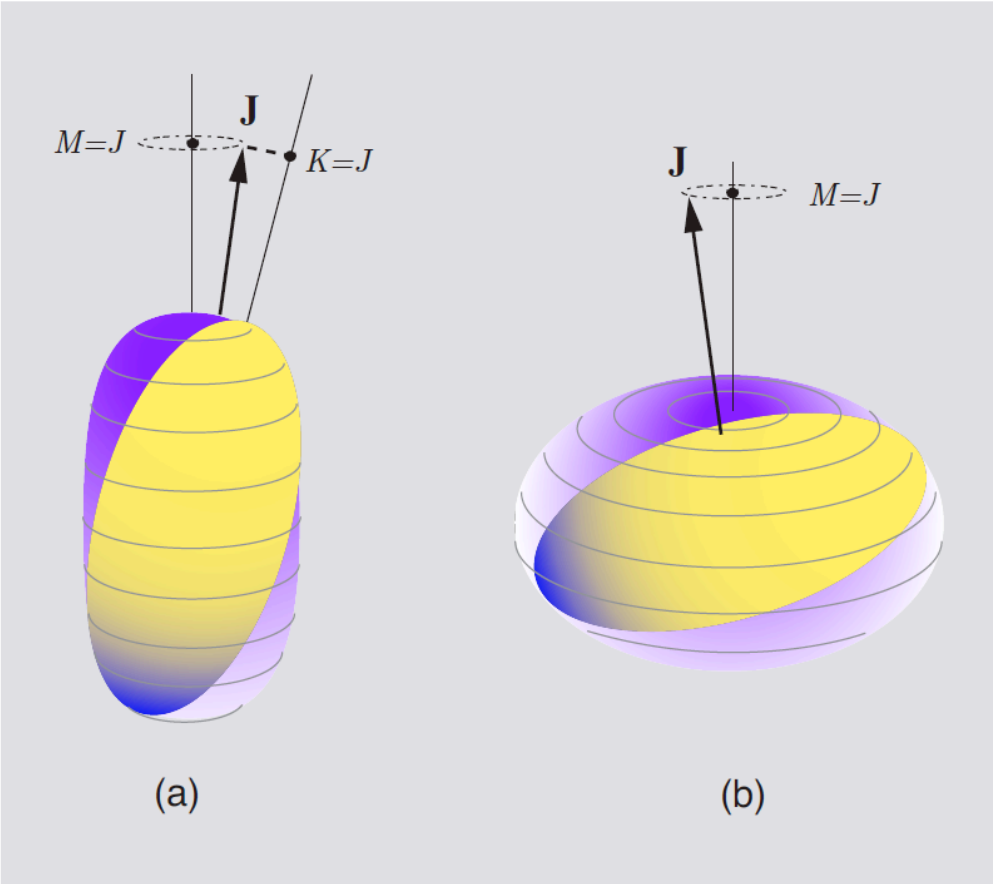
Energies of first-excited 2^+ states in nuclei

$E(2_1^+) \sim (\text{mom. of inertia})^{-1}$



R&W Fig. 1.36

Nuclear rotation—
in the laboratory frame nuclei look “fuzzy”

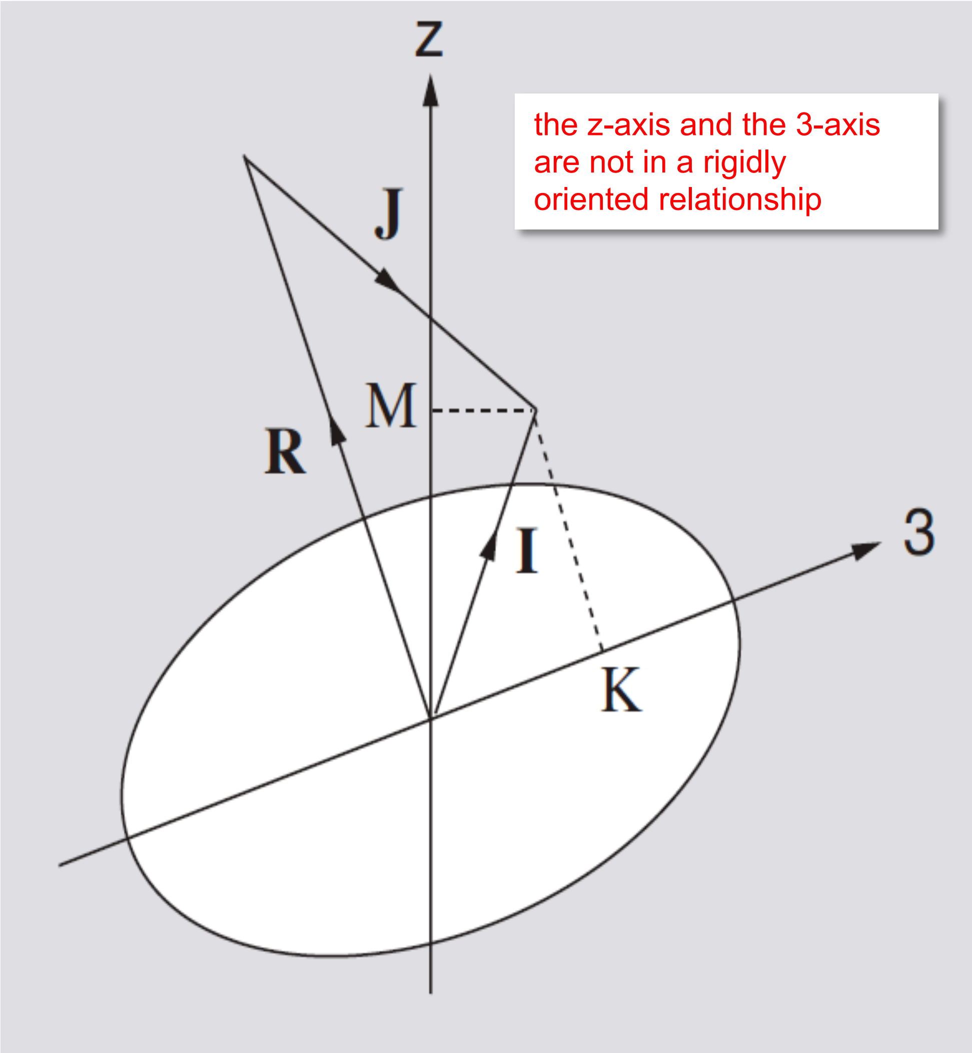


I or J are used interchangeably

Prolate rotor: (a) K = J (b) K = 0

R&W Fig. 1.45

Axially symmetric rigid rotor model: quantum numbers



the z-axis and the 3-axis are not in a rigidly oriented relationship

R: collective angular momentum

J: intrinsic spin

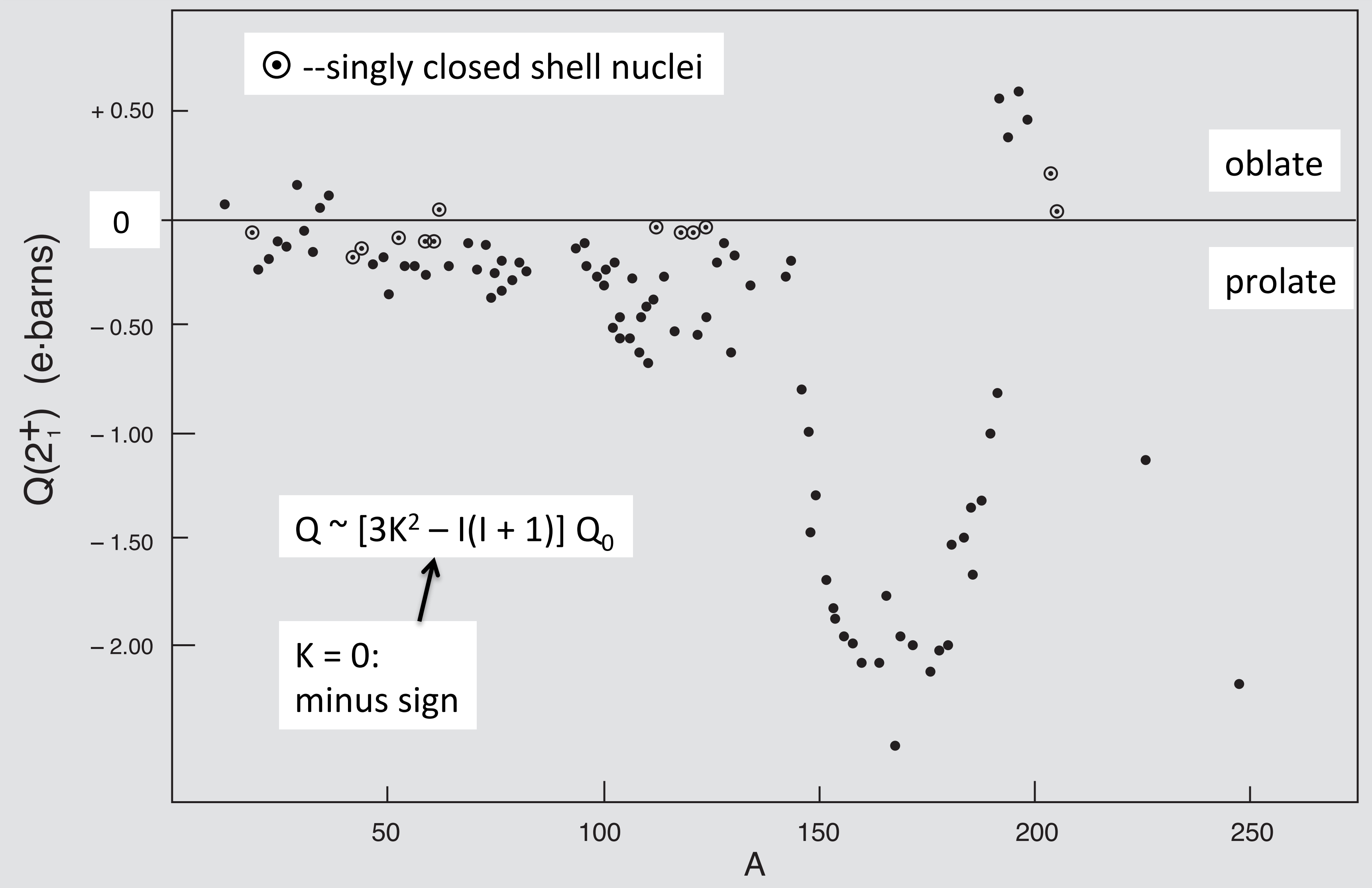
I: total spin / angular momentum

M: laboratory-frame,
z- component of I

K: body-frame (symmetry axis),
3-component of I

R&W Fig. 1.46

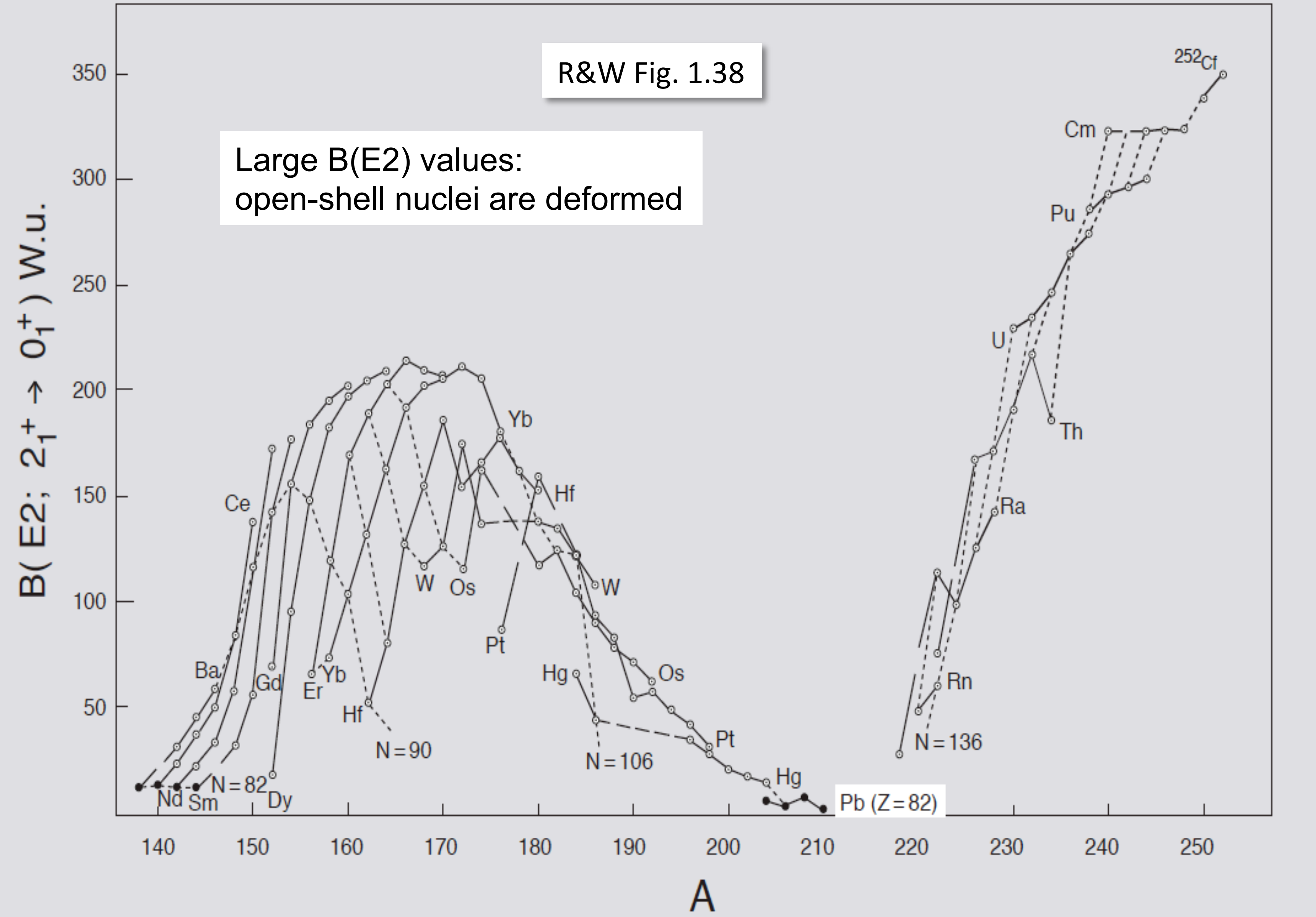
Quadrupole moments of 2^+, K = 0 states: most are prolate, but appear to oblate



Uncertainties? Lack of data?

R&W Fig. 1.37

Electric quadrupole transition probabilities $B(E2; 2_1^+ \rightarrow 0_1^+)$: Deformation



B(E2) values

$$B(E2) = 9527 / E_\gamma^{5}\ T_{1/2}\ A^{4/3}$$

E_γ in MeV

$T_{1/2}$ in ps*

B(E2) in Weisskopf units (W.u.)

B(E2) W.u. = $5.940 \times 10^{-6} A^{4/3}\ e^2\, b^2$

*There are multiple processes per decay path, e.g., γ decay and internal conversion; sometimes more than one decay path: $T_{1/2} = T_{1/2}$ (measured) / branching fraction.

e—unit of electrical charge; b = barns, 1b = 10^{-24} cm^2

Figure 2.4

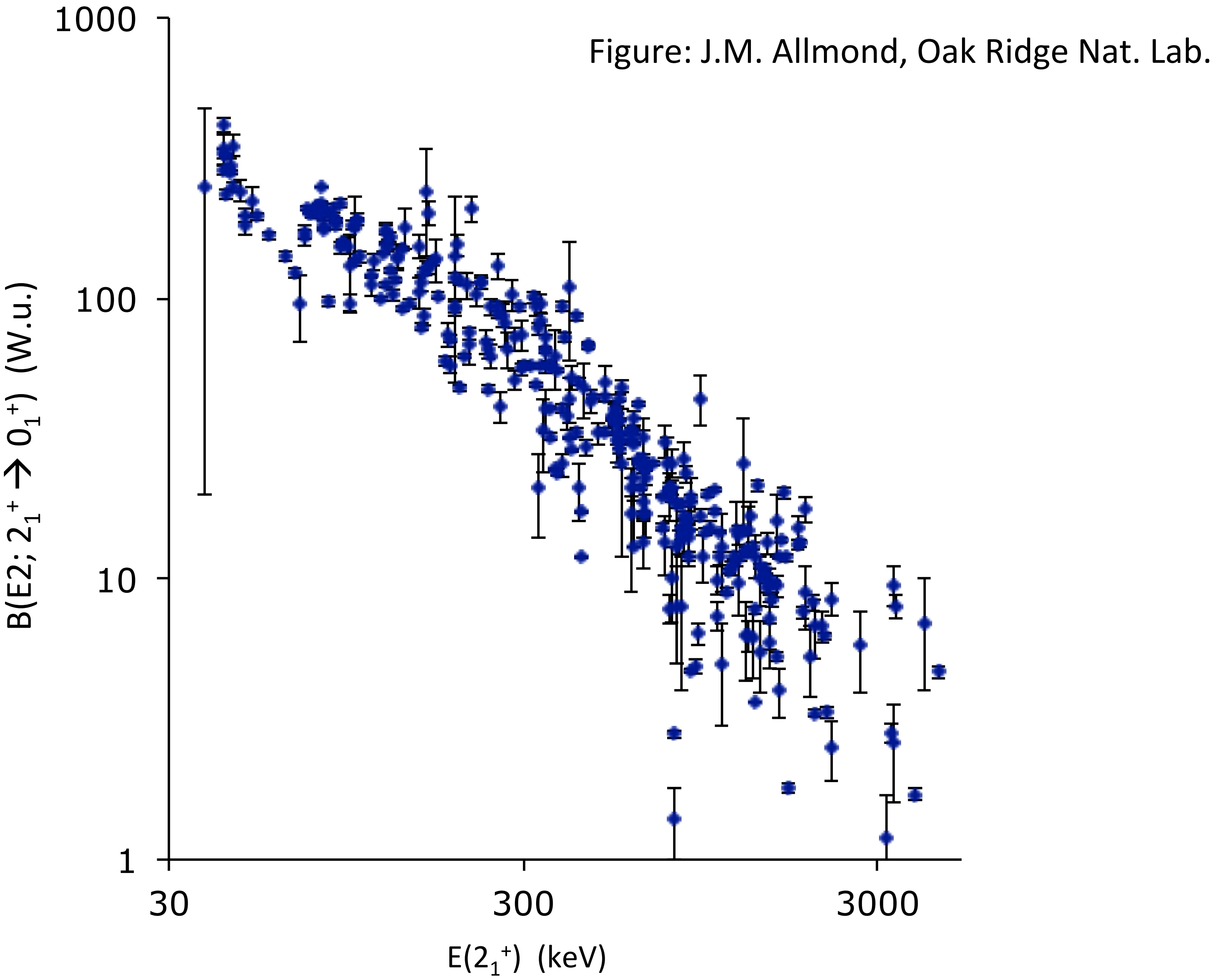


Figure 2.4. Plot of B(E2; $2_1^+ \rightarrow 0_1^+$) in W.u. versus E(2_1^+) in keV for all available data (for doubly even nuclei). This illustrates the inverse relationship between the two quantities.

Nuclear Data

- Nuclear Data Sheets
- Evaluated Nuclear Structure Data Files (ENSDF)
  nndc.bnl.gov
- Experimental Unevaluated Nuclear Data List (XUNDL)
  nndc.bnl.gov
- Masses--Atomic Mass Evaluation (AME)
  AME2016 Chin. Phys. **C41** No. 3 030001 et seq. (2017)
- Radii
  At. Data Nucl. Data Tables **99** 69 (2013)
- Moments
  At. Data Nucl. Data Tables **90** 75 (2005) [incomplete];
  ibid. **111/112** 1 (2016) [incomplete]
- B(E2) data
  At. Data Nucl. Data Tables **78** 1 (2001)
  At. Data Nucl. Data Tables **107** 1 (2016) [some errors; nonstandard eval.]

LECTURE 1: DISCUSSION

Some questions

- In the first atomic bomb detonation, how many grams of mass were converted to energy?
- How many kilograms of mass is the Sun "burning" per second?
- Why do the S_{2n} vs. A slopes for the three shell regions shown for the Ca isotopes exhibit a "steep"-"shallow"-"steep" pattern? [DIFFICULT—clue, look at a shell model diagram. Also, think of nuclear size (confining potential)]
- Why do light nuclei (^{16}O, ^{40}Ca) not quite follow the $R \sim 1.2\ A^{1/3}$ fm relationship?
- What is happening with $\delta\langle r^2\rangle$ for the Rb, Sr and Y isotopes at N = 60?
- Why does the atomic hyperfine splitting due to the electric quadrupole moment of the nucleus depend on the second derivative of the electric field?

Some nuclei are too short-lived to be isolated in the laboratory: the drip lines have been reached

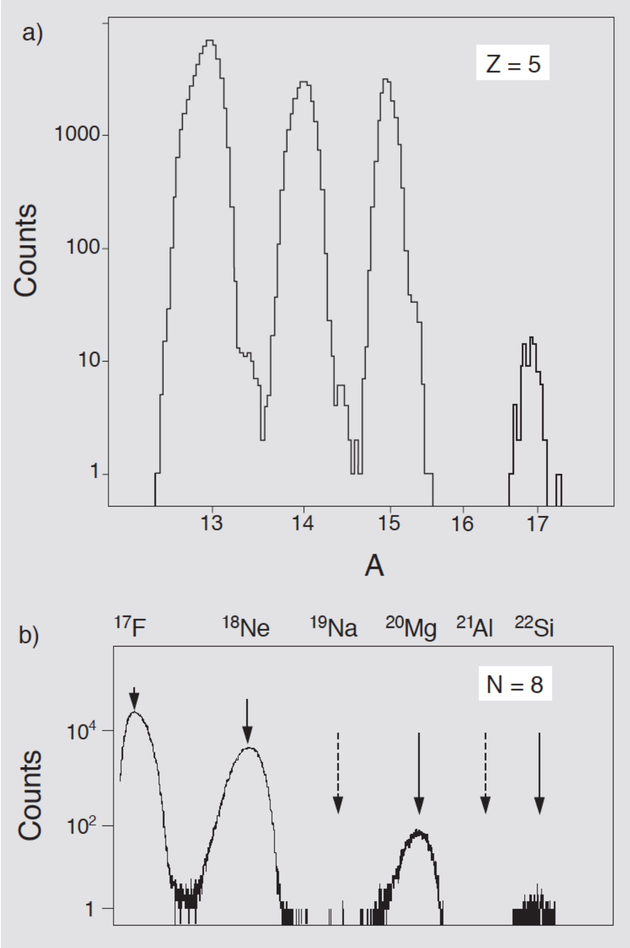
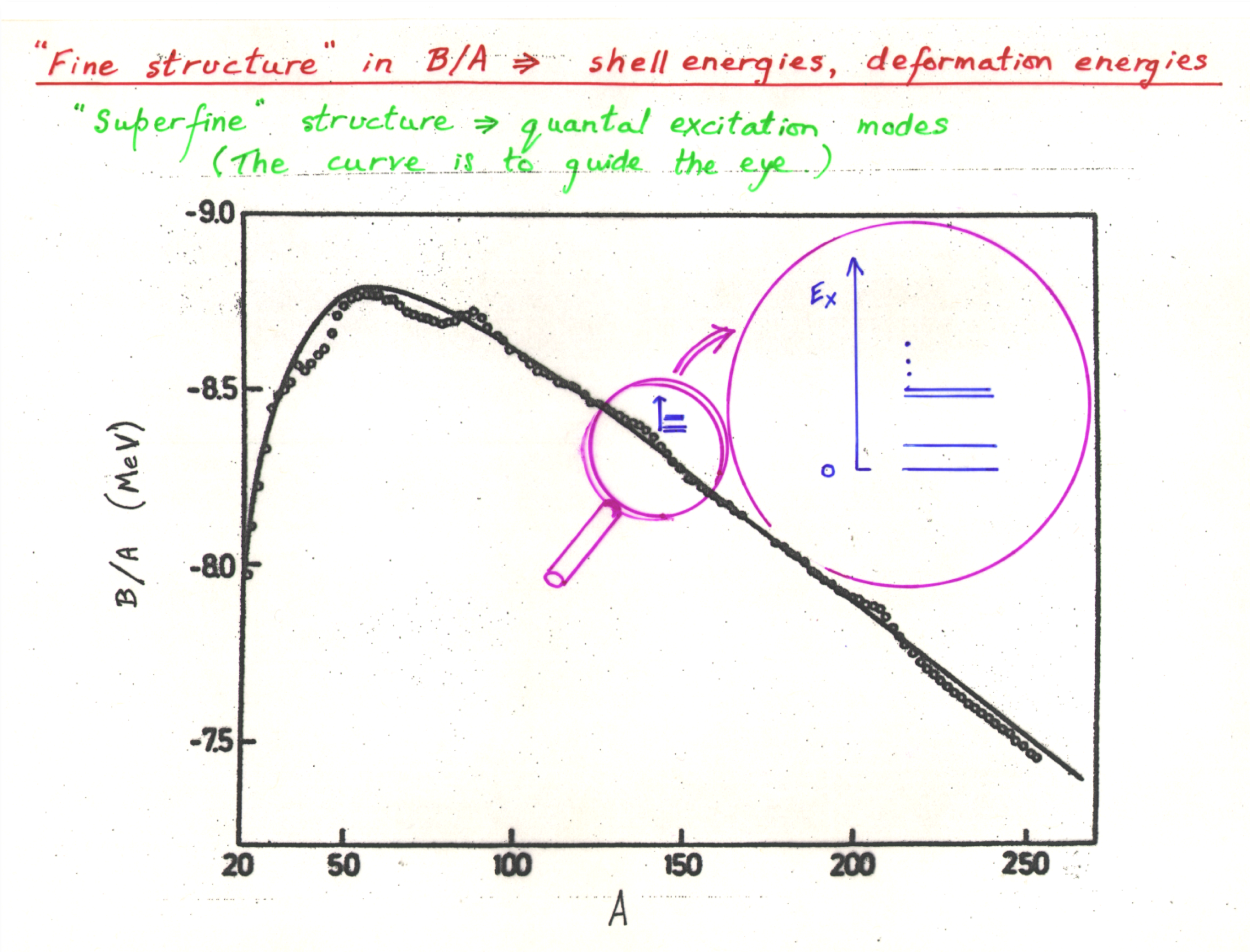


Figure a) from M. Langevin et al., PL B150, 71 (1985).

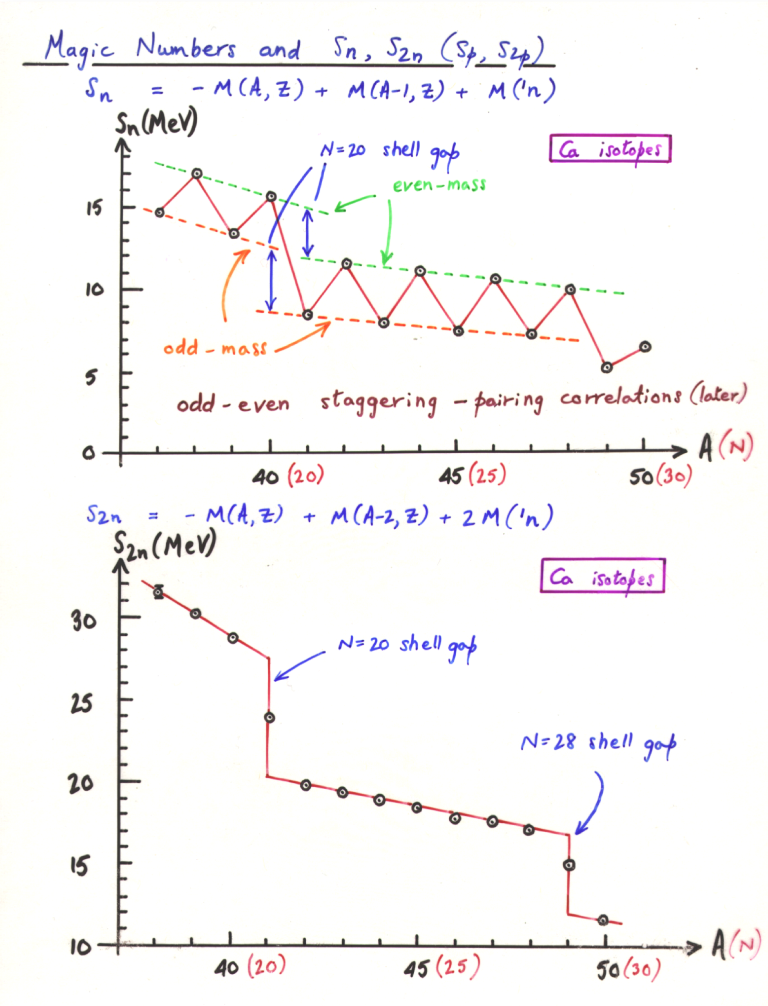
Figure b) from M.G. Saint-Laurent et al., PRL 59, 33 (1987).

R&W Fig. 1.3

The challenge to the nuclear structure physicist is to study the "fine and superfine" structure of the B/A curve



Relationship of separation energies to nuclear masses



Ground states of all even-even nuclei:

they have spin 0 because the pairing force favors maximum overlap of the nucleon wave functions;

parity is, by convention, positive for these states.

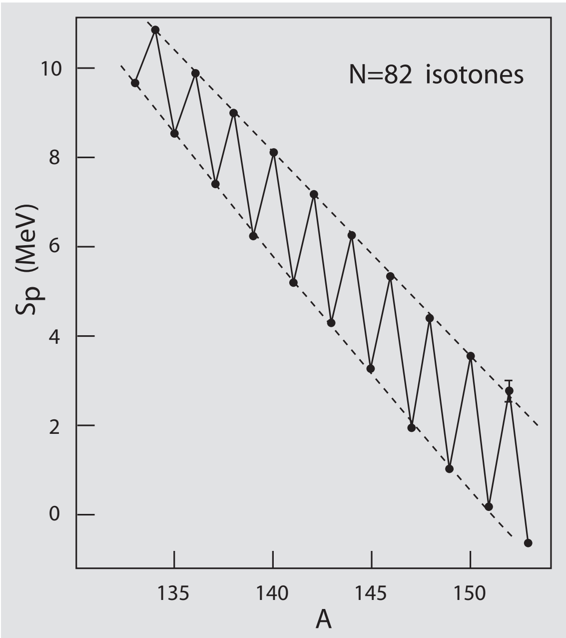
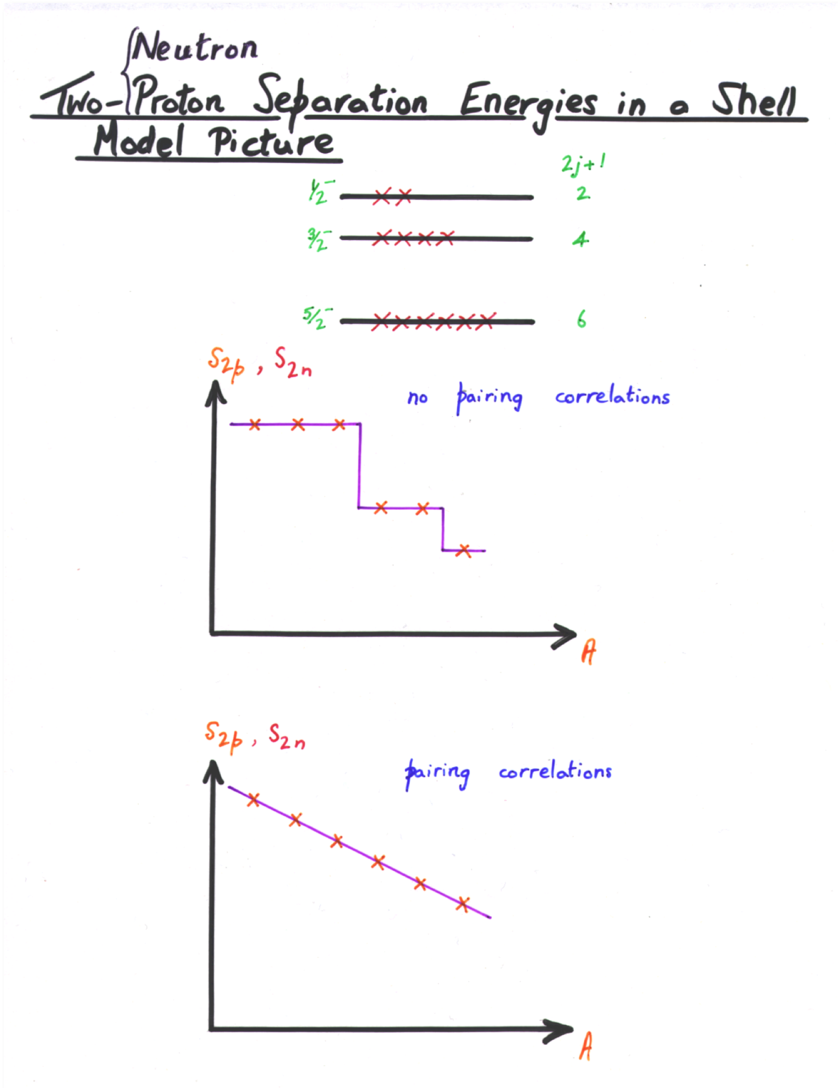


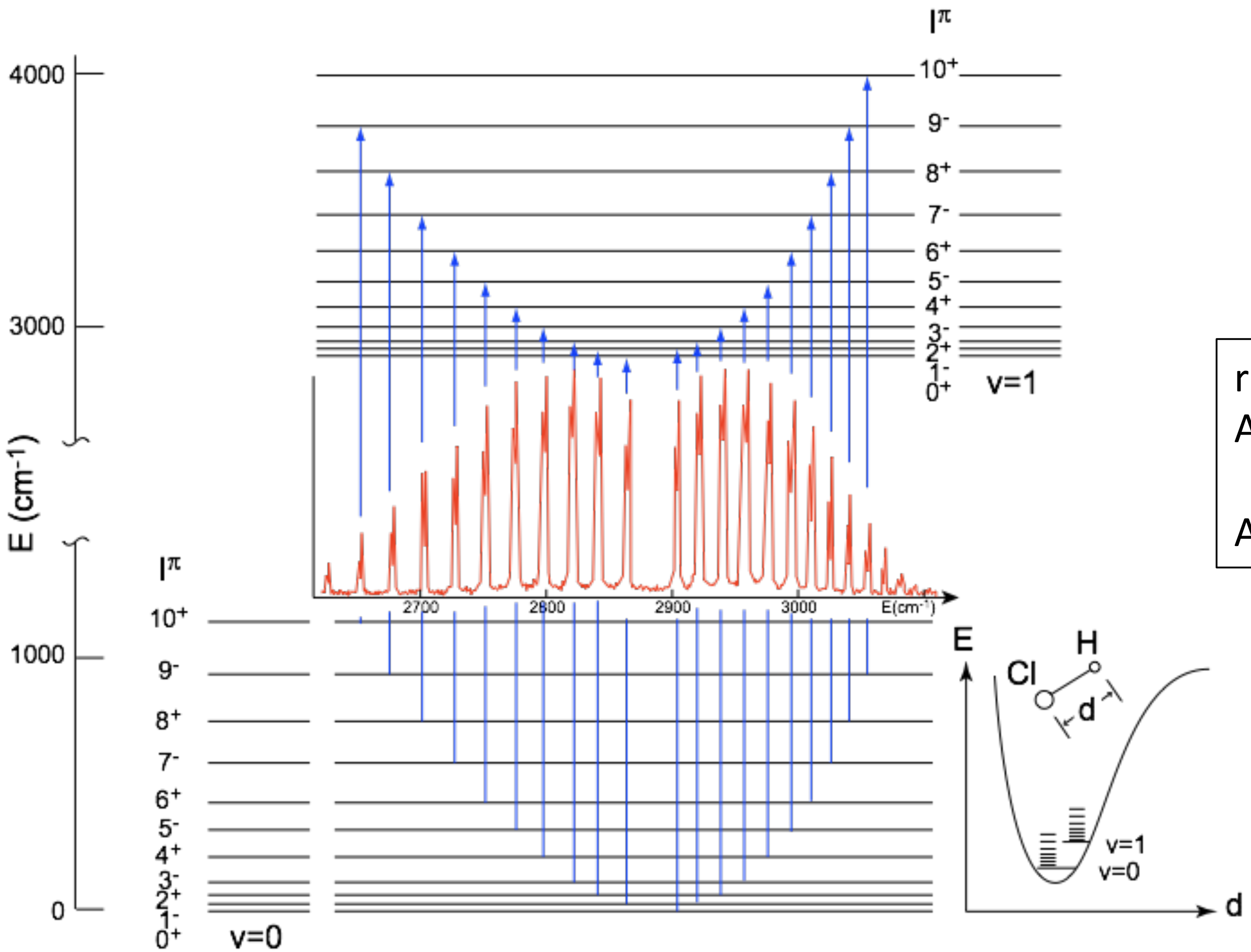
Figure 6.21: One-proton separation energies, S_p, for the $N = 82$ isotones. All lines are drawn to guide the eye. All uncertainties but one are smaller than the circles around the data points. Note that the extreme right-hand data point (for ^{153}Lu) corresponds to this nucleus being unbound with respect to one-proton emission. (Data taken from Audi G., Wapstra A.H. and Thibault C. (2003), *Nucl. Phys.* **A729**, 337.)

The anatomy of pairing in finite many-body quantum systems: effect on separation energies



In molecules, quantum degrees of freedom have separate spectral responses

The infrared absorption spectrum of HCl reveals molecular vibrations and rotations.



rotational const.
A_1 = 10.134 cm^{-1}

A_0 = 10.437 cm^{-1}

Quantum mechanics of E2 transitions and moments for the rigid axially symmetric rotor

Electromagnetic transitions and moments for the rotor

E2 transitions:

$$B(E2)\downarrow = \frac{0.056579}{(E_\gamma)^5\, T^\gamma_{1/2}}\, e^2b^2, \qquad 1\,\text{W.u.} = 5.940\times 10^{-6} A^{4/3}\, e^2b^2$$

E_γ: MeV, $T^\gamma_{1/2}$: ps; W.u.: Weisskopf unit

$$B(E2) = \frac{|\langle f|\mathcal{M}(E2)|i\rangle|^2}{2I_i+1}, \qquad B(E2;0_1^+\to 2_1^+) = 5B(E2;2_1^+\to 0_1^+)$$

$$B(E2;\alpha_i I_i K_i \to \alpha_f I_f K_f) = \frac{5}{16\pi}\langle I_i K_i\, 2, K_f-K_i | I_f K_f\rangle^2 \langle \alpha_f K_f|\hat{Q}_{K_f-K_i}|\alpha_i K_i\rangle^2, \qquad \langle \alpha_f K_f|\hat{Q}_{K_f-K_i}|\alpha_i K_i\rangle^2 \equiv Q_0^2$$

(from Wigner-Eckart theorem)

For $K=0$ ground-state bands of doubly-even nuclei:

$$B(E2;I\to I-2) = \frac{15}{32\pi}\, Q_0^2\, \frac{I(I-1)}{(2I-1)(2I+1)}, \qquad B(E2;2\to 0) = \frac{Q_0^2}{16\pi}$$

Electric quadrupole moments:

$$Q_{\alpha K}(I) \equiv \langle \alpha K I M=I|\hat{Q}_0|\alpha K I M=I\rangle$$

$$Q_K(I) = Q_0\left[\frac{3K^2 - I(I+1)}{(I+1)(2I+3)}\right]$$

For $K=0$ gsb's e-e:

$$Q(I) = -\frac{I}{2I+3}\, Q_0$$

$$Q(2_1^+) = -\frac{2}{7}\sqrt{16\pi\cdot B(E2;2_1^+\to 0_1^+)}$$