# LECTURE 2

## Classifying nuclear structures: mostly even mass

Key structural types

Elementary quantum mechanical descriptions

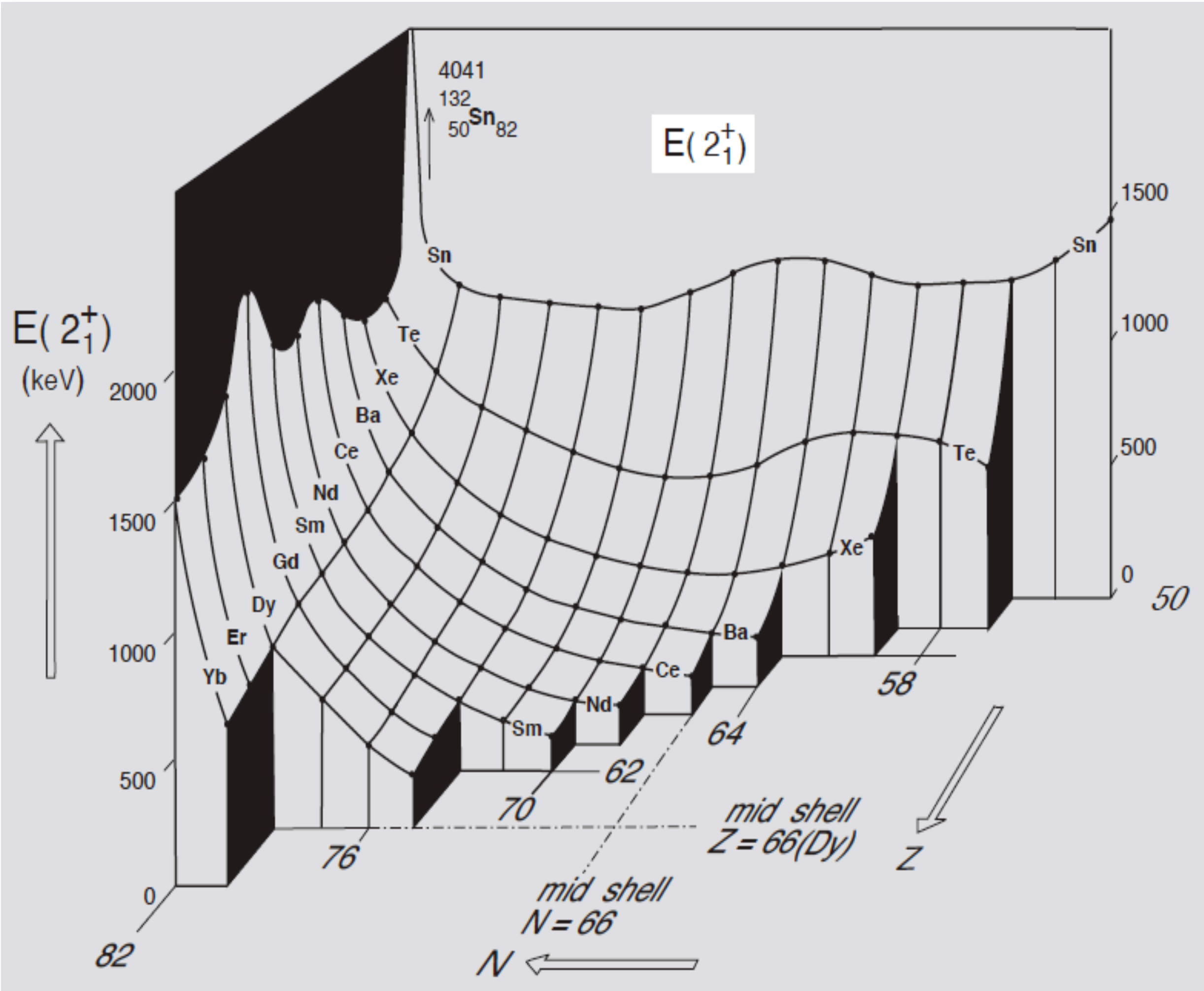
# KEY OBSERVATIONS

Given a collection of nucleons, we possess no *a priori* way to arrive at the structure of nuclei without guidance from data.

Given a collection of data, we possess no *a priori* way to arrive at the structure of nuclei without guidance from quantum mechanical models.

# Energies of first-excited $2^+$ states in nuclei

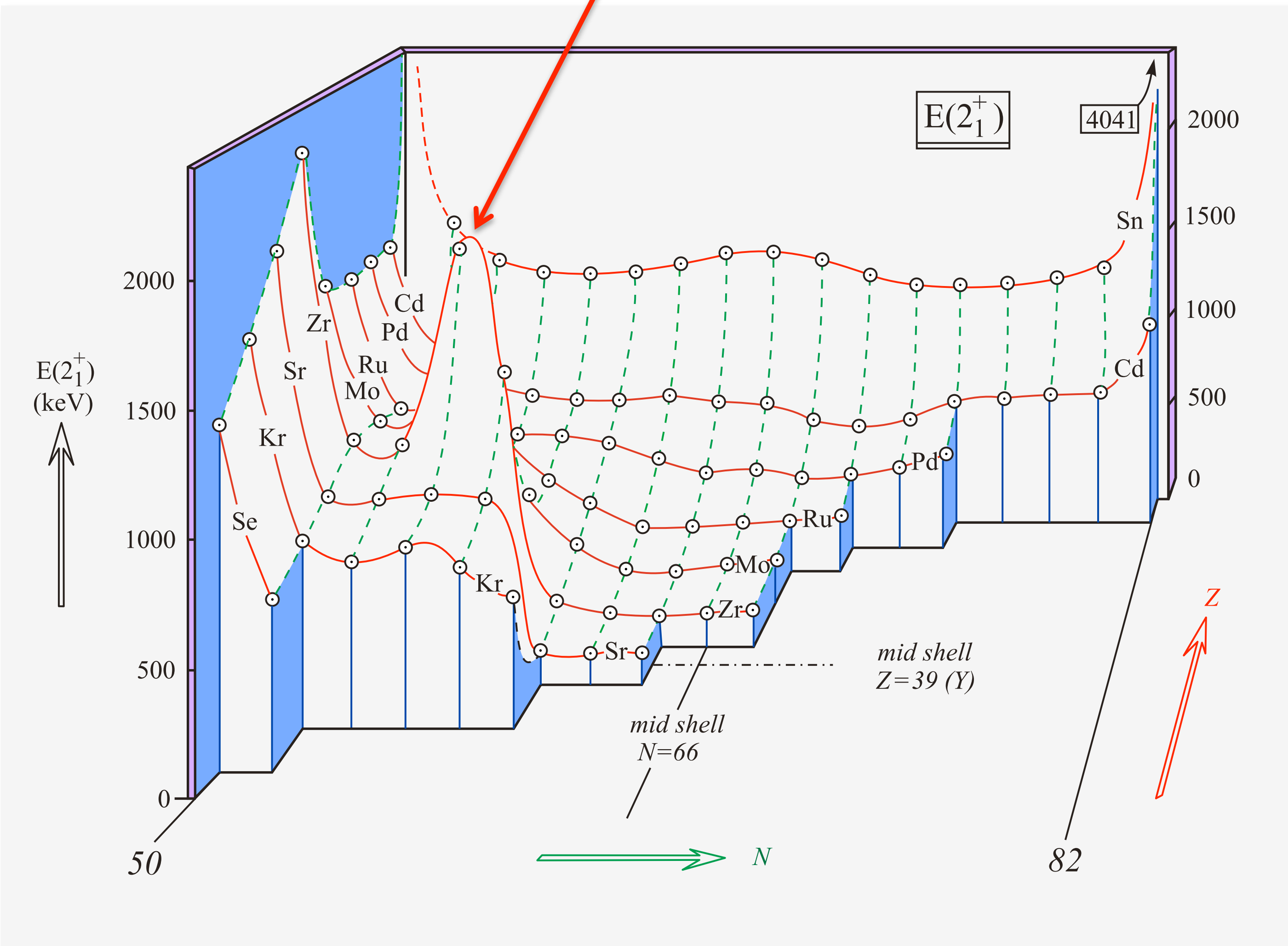
$E(2_1^+) \sim (\text{mom. of inertia})^{-1}$




R&W Fig. 1.36

# Systematic of $E(2_1^+)$ for $N \geq 50$, $Z \leq 50$

$^{96}$Zr

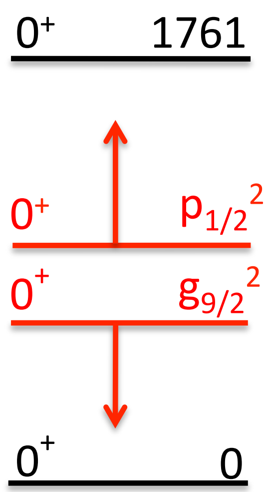


# High energies for $2_1^+$ states may be misleading

**N=50: $g_{9/2}$ seniority structure, v = 0, 2**

**J = ½ orbitals can only contribute to v = 0 states, at low energy**

**$^{90}$Zr E($2_1^+$) is high: suggests a closed subshell, but is due to *depression* of the ground-state energy**



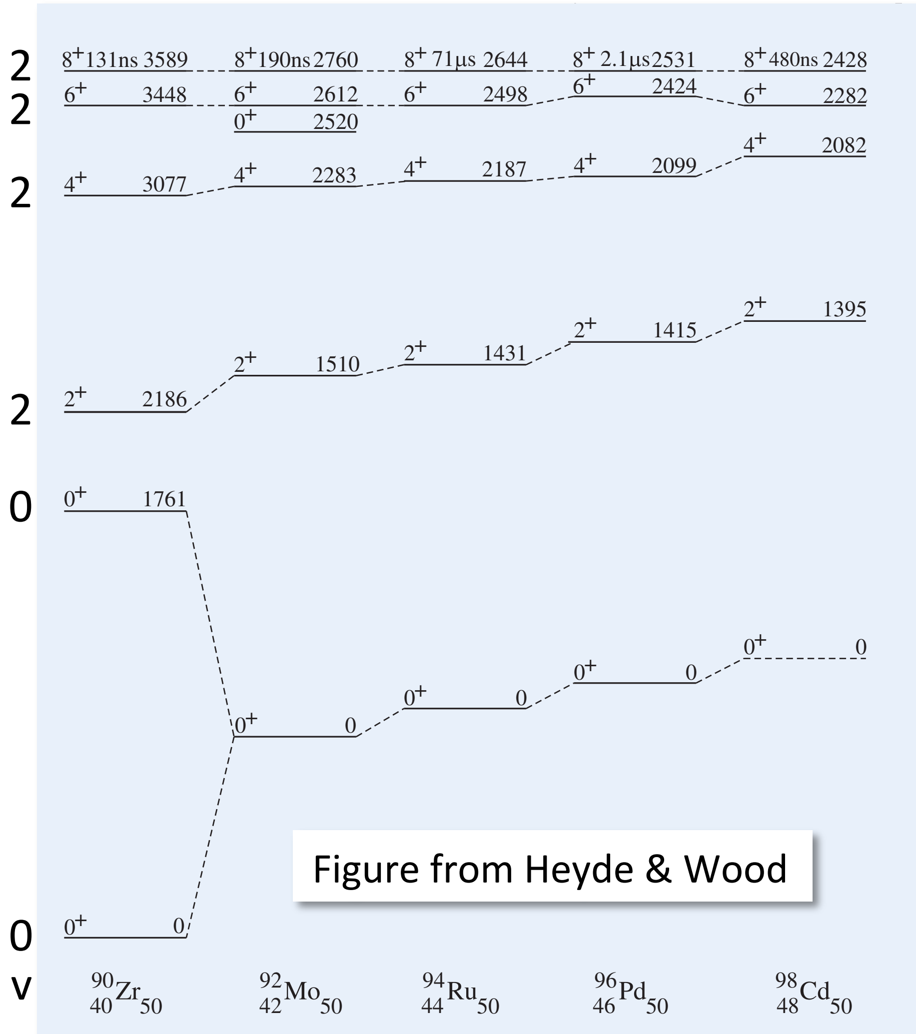




Figure from Heyde & Wood

# $E(2_1^+)$ systematic: a simple view of nuclear structure

Figure from Heyde & Wood

| | | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Cr | 24 | | | 892 | 752 | 783 | 1434 | 835 | 1007 | 881 | 646 |
| Ti | 22 | | 1556 | 1083 | 889 | 983 | 1554 | 1050 | 1495 | 1129 | |
| Ca | 20 | 2213 | 3904 | 1525 | 1157 | 1346 | 3832 | 1026 | 2563 | | |
| Ar | 18 | 1970 | 2168 | 1461 | 1208 | 1158 | 1577 | 1037 | | | |
| S | 16 | 2127 | 3291 | 1292 | 904 | 890 | 1330 | 952 | | | |
| Si | 14 | 1941 | 3328 | 1399 | 1084 | 986 | 770 | | | | |
| Mg | 12 | 1483 | 886 | 660 | 660 | | | | | | |
| Ne | 10 | 1320 | 792 | 722 | | | | | | | |

$B_n \sim 0$

$E(2_1^+)$

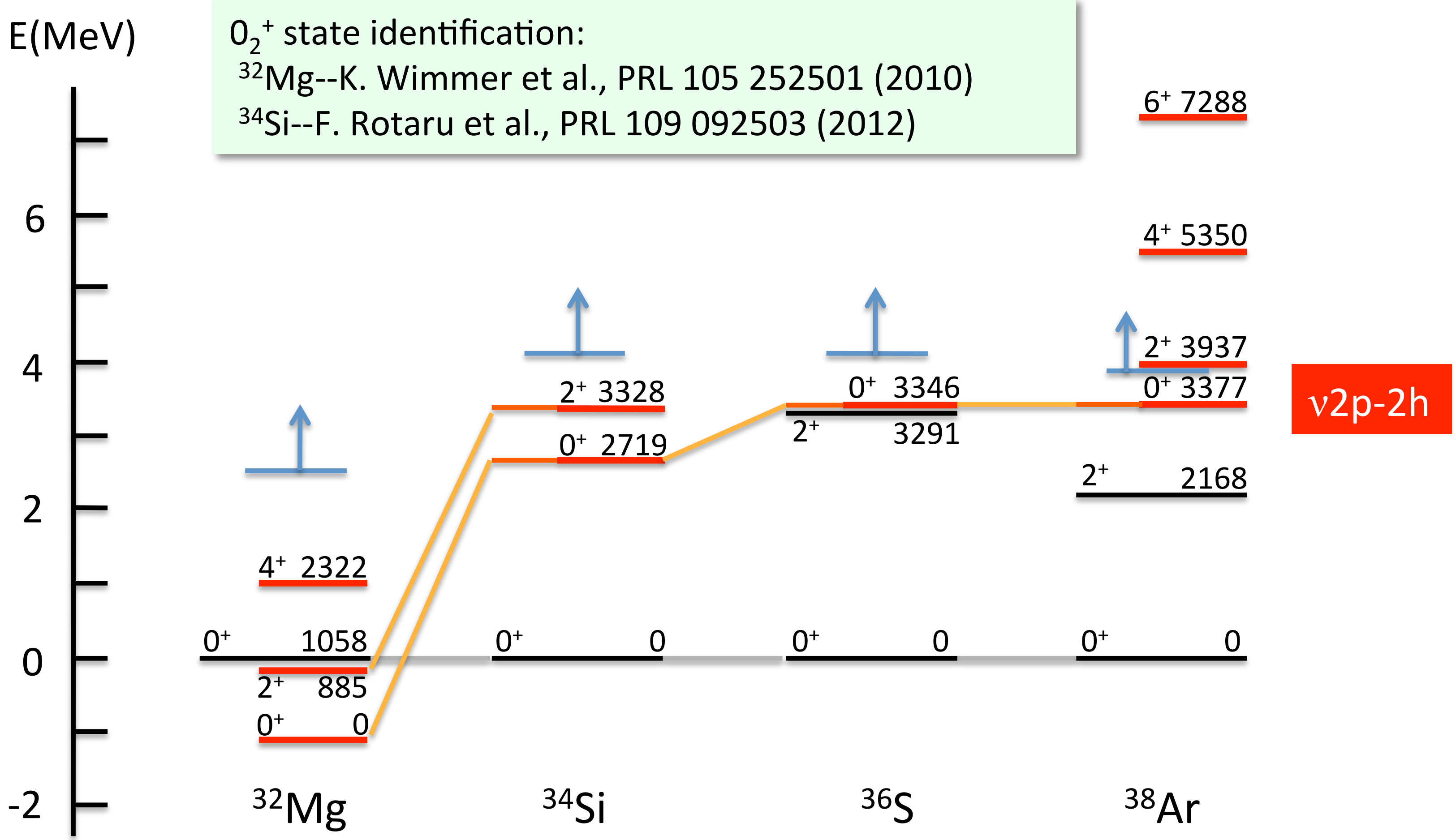
Has the shell structure @ N=20 "collapsed" or "melted" for Z ≤ 12?

And @ N=28 for Z ≤ 14?

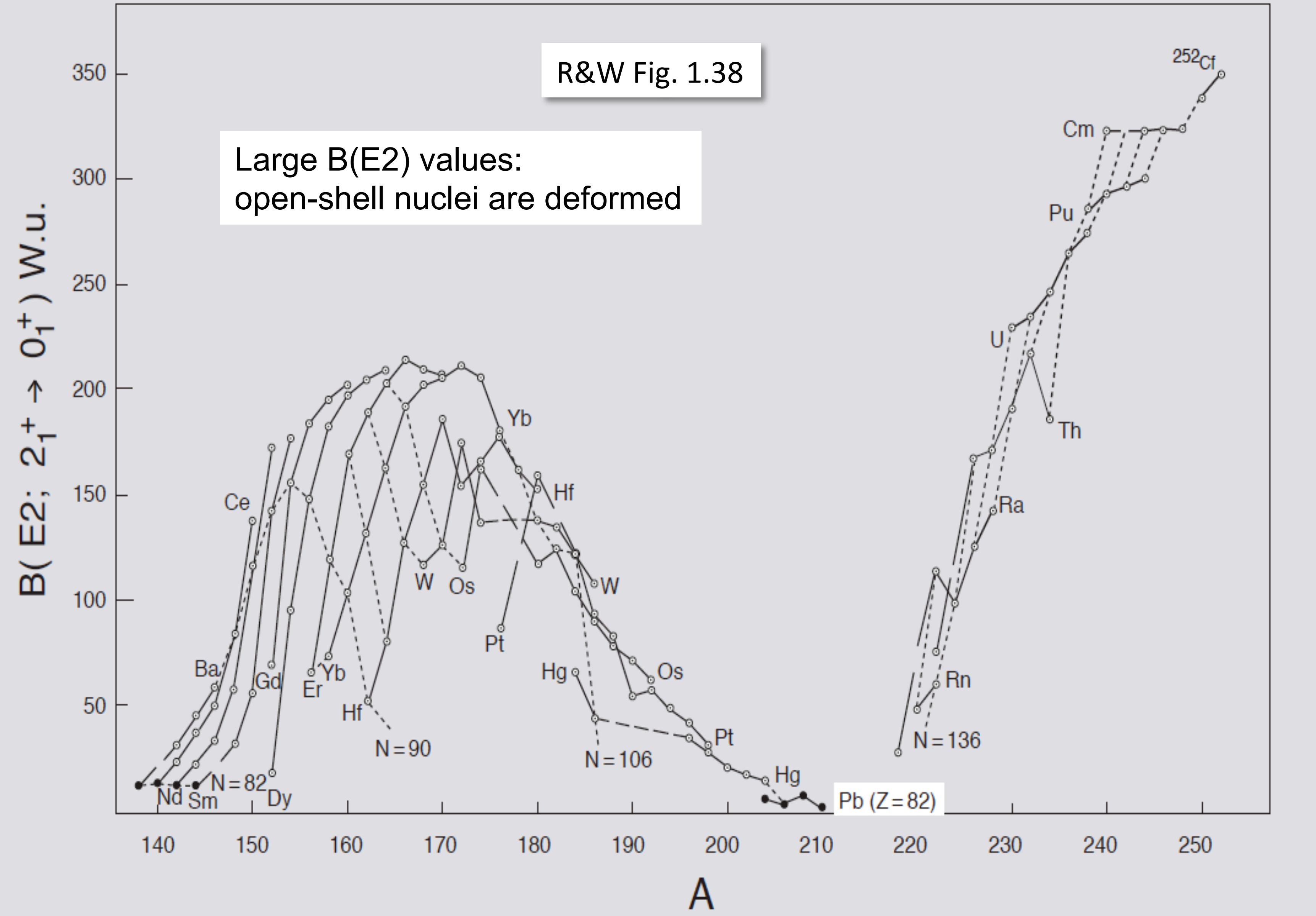
Energies are in keV

# Intruder states or
# the "island of inversion" @ N=20

$0_2^+$ state identification:
$^{32}$Mg--K. Wimmer et al., PRL 105 252501 (2010)
$^{34}$Si--F. Rotaru et al., PRL 109 092503 (2012)



# Electric quadrupole transition probabilities $B(E2; 2_1^+ \to 0_1^+)$: Deformation



# B(E2) values

$$B(E2) = 9527 / E_\gamma^{5}\ T_{1/2}\ A^{4/3}$$

$E_\gamma$ in MeV

$T_{1/2}$ in ps*

B(E2) in Weisskopf units (W.u.)

B(E2) W.u. = $5.940 \times 10^{-6} A^{4/3}\ e^2 b^2$

*There are multiple processes per decay path, e.g., $\gamma$ decay and internal conversion; sometimes more than one decay path: $T_{1/2} = T_{1/2}$ (measured) / branching fraction.

e—unit of electrical charge; b = barns, 1b = $10^{-24}$ $cm^2$

# V.F. Weisskopf (units): Phys. Rev. **83** 1073 (1951)



where $K$ is the low frequency dielectric constant, $K_0$ is the optical constant, $\rho$ the density, and $\chi$ the compressibility. In Table I are listed the values of $\partial \ln K/\partial p$ calculated from (4) and (1) next to the experimental values of $\partial \ln K/\partial p$. The calculated values of $\partial \ln K/\partial p$ differ from those of Rao by the term $a(K-K_0)/K$, which arises from the difference between (1a) and (2a).

Equation (4) is derived assuming that the inner field polarizing the dielectric is independent of pressure. Since the values of $-\partial \ln K/\partial p$ obtained from (4) do not account for all the change in the dielectric constant, it seems consistent to expect that the inner field is not constant but does decrease with increasing pressure. This conclusion agrees with the one reached in my original paper using the theories of Hojendahl and Mott and Littleton.

[1] D. A. A. S. Narayana Rao, Phys. Rev. **82**, 118 (1951).
[2] S. Mayburg, Phys. Rev. **79**, 375 (1950).

## Radiative Transition Probabilities in Nuclei

V. F. Weisskopf
*Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts*
(Received July 20, 1951)

CONSIDER a transition from nuclear state $a$ to nuclear state $b$ with emission of a quantum of multipole radiation of angular momentum $l$ ($2^l$-pole) and $z$ component $m$. The transition probability per unit time is given by[1]

$$T(l,m)=\frac{8\pi(l+1)}{l[(2l+1)!!]^2}\frac{\kappa^{2l+1}}{\hbar}|A(l,m)+A'(l,m)|^2, \quad (1)$$

where $\kappa=2\pi\nu/c$ is the wave number of the emitted radiation, and the quantities $A$, $A'$ are the multipole matrix elements caused by the electric currents and by the magnetization (spins), respectively. We find for electric radiation

$$A(l,m)=Q(l,m)=e\sum_{k=1}^{Z}\int r_k{}^l Y_{lm}{}^*(\theta_k,\phi_k)\,\varphi_b{}^*\varphi_a\,d\tau, \quad (2)$$

$$A'(l,m)=Q'(l,m)=-\frac{i\kappa}{l+1}\frac{e\hbar}{2Mc}\sum_{k=1}^{A}\mu_k \times\int r_k{}^l Y_{lm}{}^*(\theta_k,\phi_k)\,\mathrm{div}(\varphi_b{}^*\mathbf{r}_k\times\boldsymbol{\sigma}_k\varphi_a)\,d\tau, \quad (3)$$

where $\varphi_a$ and $\varphi_b$ are the wave functions of the nuclear states, $M$ is the mass of each nucleon, $\mathbf{r}_k=(r_k,\theta_k,\phi_k)$ is the position vector of the $k$th nucleon, $\boldsymbol{\sigma}_k$ is its Pauli spin vector, and $\mu_k$ is its magnetic moment in nuclear magnetons. The sum in (2) extends over the protons, the sum in (3) over both protons and neutrons. These expressions are approximations valid for $\kappa R\ll 1$, where $R$ is the nuclear radius.

The corresponding expressions for magnetic multipole radiation are

$$A(l,m)=M(l,m)=-\frac{1}{l+1}\frac{e\hbar}{Mc}\sum_{k=1}^{Z} \times\int r_k{}^l Y_{lm}{}^*(\theta_k,\phi_k)\,\mathrm{div}(\varphi_b{}^*\mathbf{L}_k\varphi_a)\,d\tau, \quad (4)$$

$$A'(l,m)=M'(l,m)=-\frac{e\hbar}{2Mc}\sum_{k=1}^{A}\mu_k \times\int r_k{}^l Y_{lm}{}^*(\theta_k,\phi_k)\,\mathrm{div}(\varphi_b{}^*\boldsymbol{\sigma}_k\varphi_a)\,d\tau, \quad (5)$$

where $\mathbf{L}_k=-i\mathbf{r}_k\times\nabla_k$ is the orbital angular momentum operator (in units of $\hbar$) for the $k$th nucleon.

We can estimate these matrix elements by the following exceedingly crude method. We assume that the radiation is caused by a transition of one single proton which moves independently within the nucleus, its wave function being given by $u(r)Y_{lm}(\theta,\phi)$. In addition we also assume that the final state of the proton is an $S$ state.[2] We then obtain

$$Q(l,m)\sim[e/(4\pi)^{\frac{1}{2}}][3/(l+3)]R^l \quad (6)$$

where the integral $\int r^l u_b(r)u_a(r)r^2dr$ over the radial parts of the proton wave functions was set approximately equal to $3R^l/(l+3)$. The other matrix elements are estimated by replacing div by $R^{-1}$. We get the rough order-of-magnitude guess

$$M(l,m)\sim[e/(4\pi)^{\frac{1}{2}}][3/(l+3)][\hbar/Mc]R^{l-1}, \quad (7)$$

$$M'(l,m)\sim[e/(4\pi)^{\frac{1}{2}}][3/(l+3)]\mu_P[\hbar/Mc]R^{l-1}, \quad (8)$$

where $\mu_P$ is the magnetic moment of the proton ($=2.78$). $Q'(l,m)$ can be neglected compared to $Q(l,m)$. We therefore get a ratio of roughly

$$(1+\mu_P{}^2)(\hbar/McR)^2\sim 10(\hbar/McR)^2$$

between the transition probability of a magnetic multipole and an electric one of the same order. This ratio is energy-independent in contrast to widespread belief.

Inserting these estimates into (1) we get for the transition probability of an electric $2^l$-pole

$$T_E(l)\cong\frac{4.4(l+1)}{l[(2l+1)!!]^2}\left(\frac{3}{l+3}\right)^2\left(\frac{\hbar\omega}{197\text{ Mev}}\right)^{2l+1} \times(R\text{ in }10^{-13}\text{ cm})^{2l}\,10^{21}\text{ sec}^{-1} \quad (9)$$

and for a magnetic $2^l$-pole

$$T_M(l)\cong\frac{1.9(l+1)}{l[(2l+1)!!]^2}\left(\frac{3}{l+3}\right)^2\left(\frac{\hbar\omega}{197\text{ Mev}}\right)^{2l+1} \times(R\text{ in }10^{-13}\text{ cm})^{2l-2}\,10^{21}\text{ sec}^{-1}. \quad (10)$$

The assumptions made in deriving these estimates are extremely crude and they should be applied to actual transitions with the greatest reservations. They are based upon an extreme application of the independent-particle model of the nucleus and it was assumed that a proton is responsible for the transition. On the basis of our assumptions the electric multipole radiation with $l>1$ should be much weaker for transitions in which a single neutron changes its quantum state. No such differentiation is apparent in the data.

In spite of these difficulties it may be possible that the order of magnitude of the actual transition probabilities is correctly described by these formulas. We have published these exceedingly crude estimates only because of the rather unexpected agreement with the experimental material which was pointed out to us by many workers in this field.

The author wishes to express his appreciation especially to Dr. M. Goldhaber and Dr. J. M. Blatt for their great help in discussing the experimental material and in improving the theoretical reasoning.

[1] We use the notation $(2l+1)!!=1\cdot3\cdot5\cdots(2l+1)$.
[2] This latter assumption can be removed; the corrections consist in unimportant numerical factors.

## Nuclear Magnetic Resonance in Metals: Temperature Effects for $Na^{23}$

H. S. Gutowsky
*Noyes Chemical Laboratory, Department of Chemistry, University of Illinois, Urbana, Illinois**
(Received July 2, 1951)

KNIGHT reported[1] that nuclear magnetic resonance frequencies are higher in metals than in chemical compounds. It has been proposed[2] that such frequency shifts are primarily the result of the contribution of conduction electrons to the magnetic field at the nuclei in the metal. This note gives an account of some related preliminary results including temperature and chemical effects, and also detailed line shape studies. Our experiments have been at fixed frequency using equipment and procedures outlined previously.[3,4]

The effect of temperature on the $Na^{23}$ magnetic resonance shift in the metal, relative to a sodium chloride solution, is given in

The assumptions made in deriving these estimates are extremely crude and they should be applied to actual transitions with the greatest reservations.

Figure 2.4

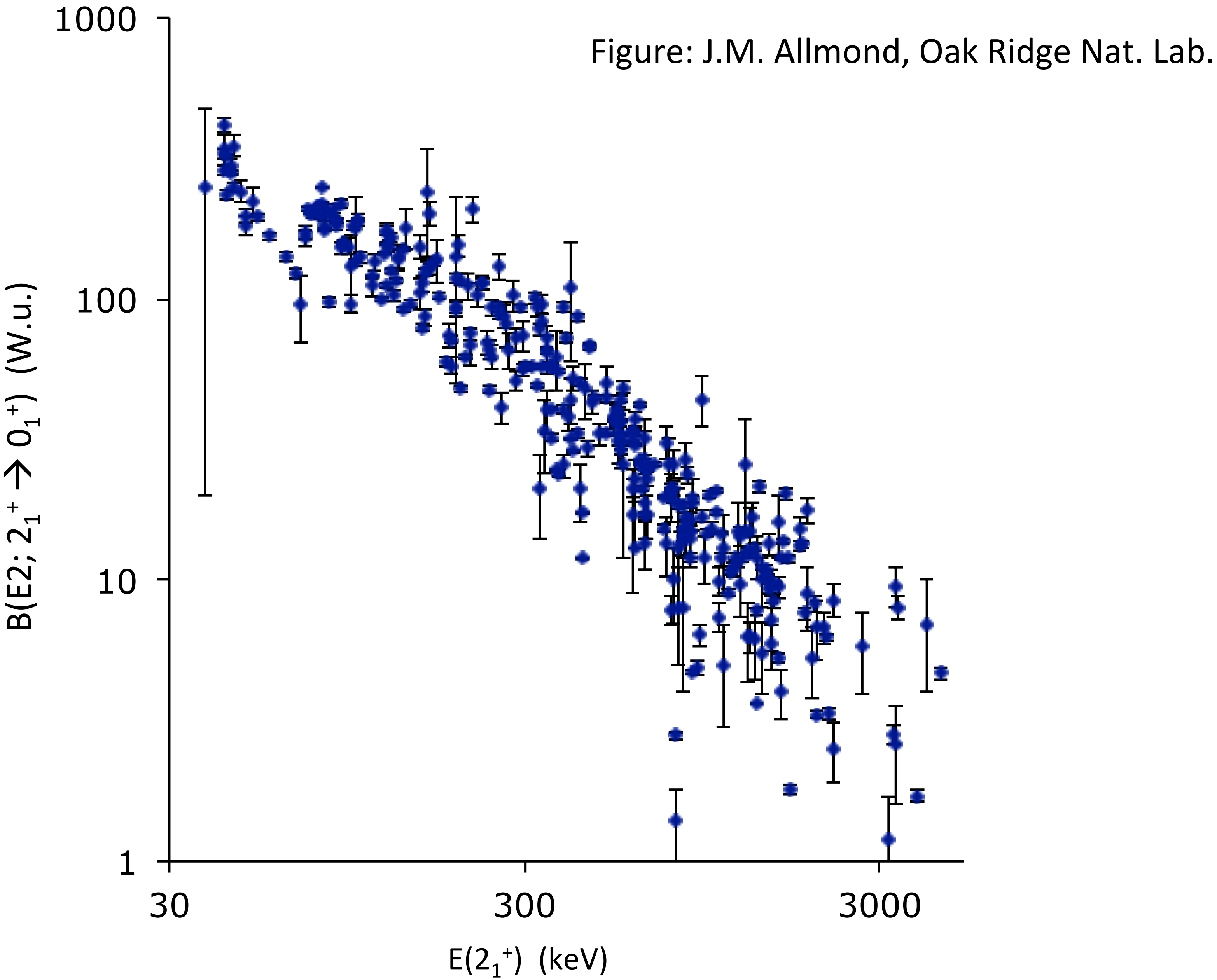
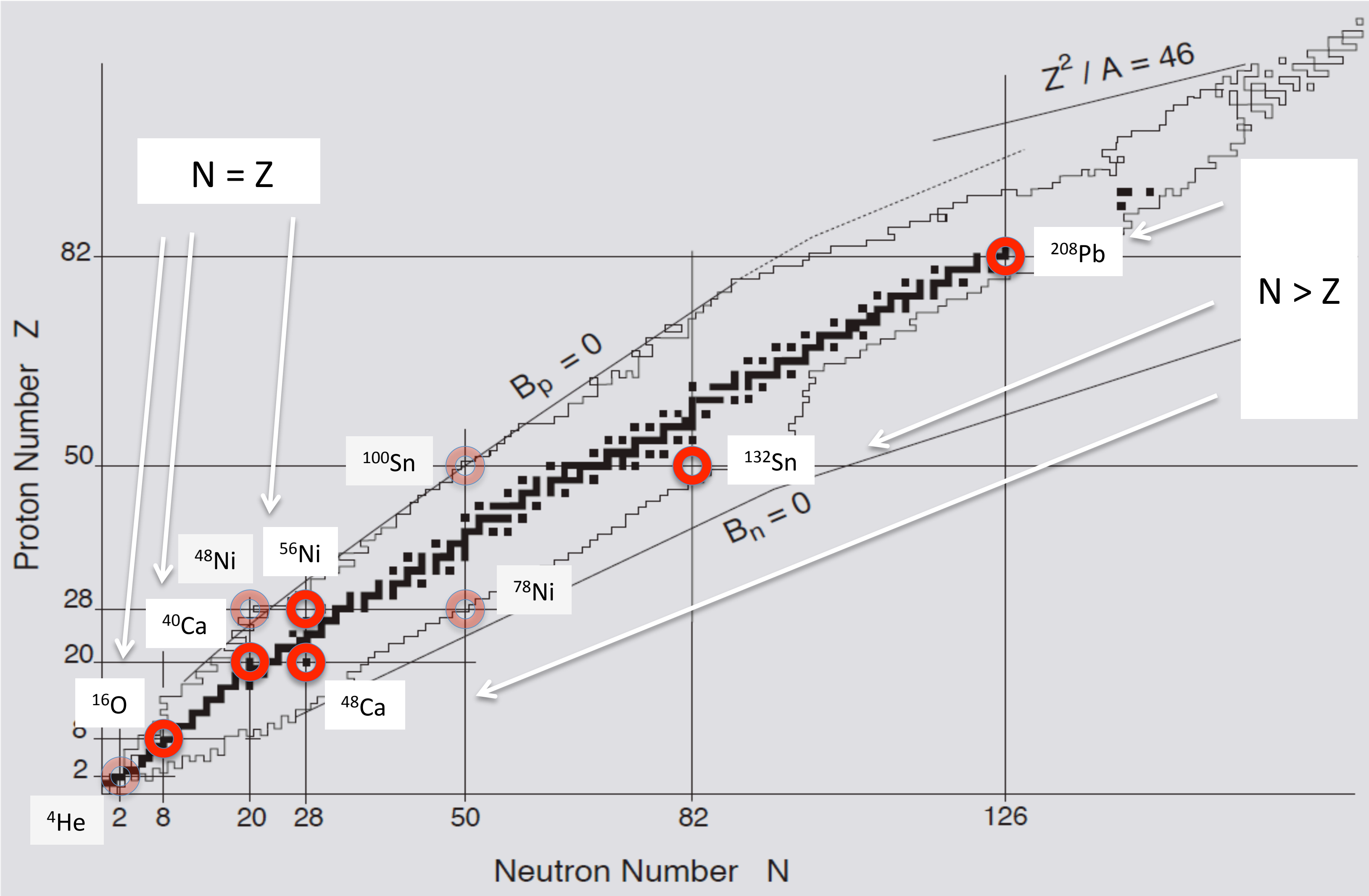



Figure 2.4. Plot of B(E2; $2_1^+ \rightarrow 0_1^+$) in W.u. versus E($2_1^+$) in keV for all available data (for doubly even nuclei). This illustrates the inverse relationship between the two quantities.

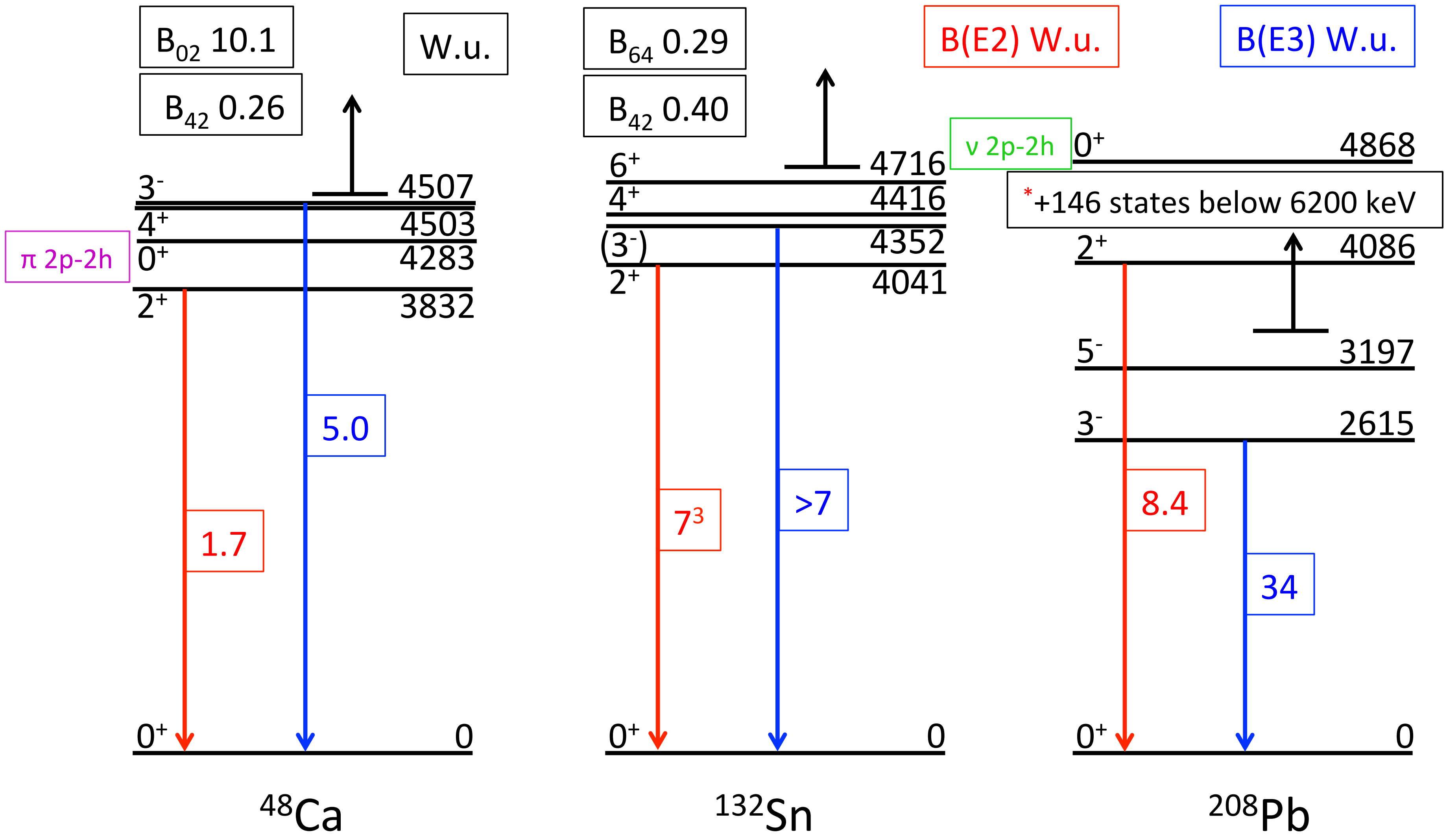
# Doubly closed shell nuclei




R&W Fig. 1.2

# Doubly closed shells: $^{48}$Ca, $^{132}$Sn, $^{208}$Pb*

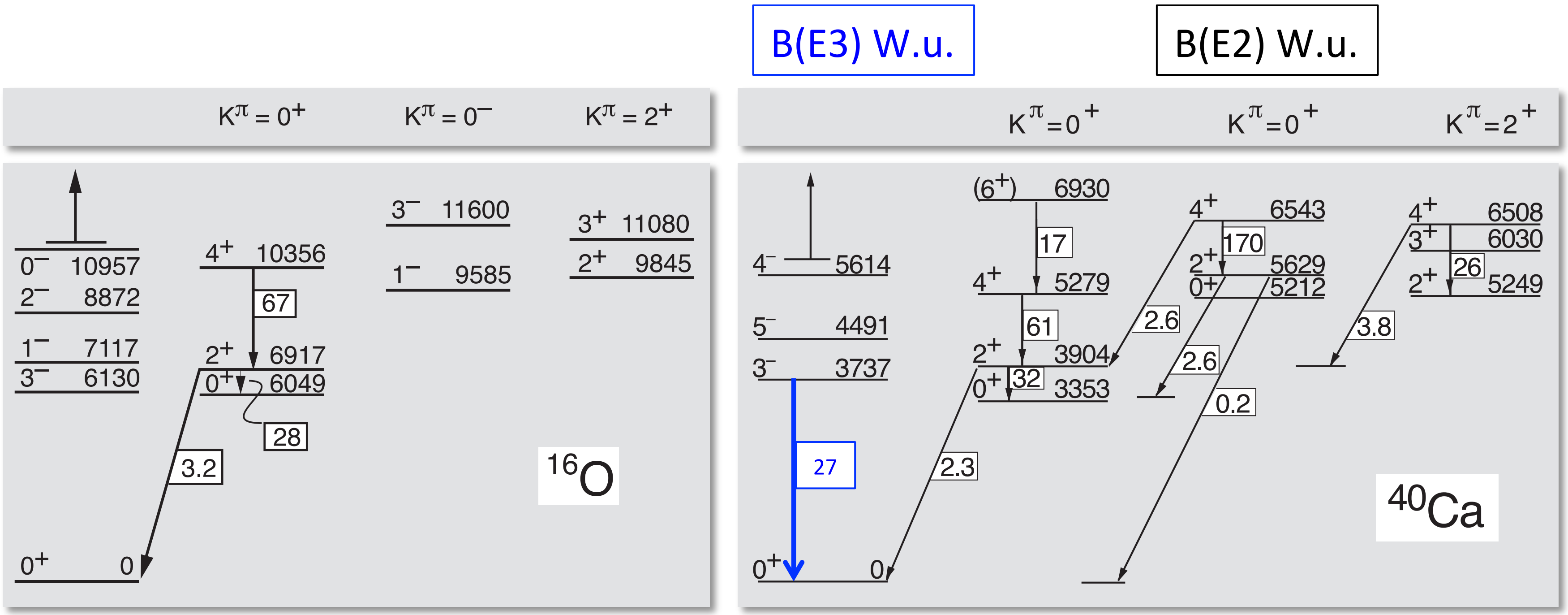
*See: A. Heusler et al., PR C **93** 054321 (2016)



# Doubly closed shells, N = Z: $^{16}O$, $^{40}Ca$

Doubly closed shell nuclei with N = Z exhibit shape coexistence at (relatively) low energy.

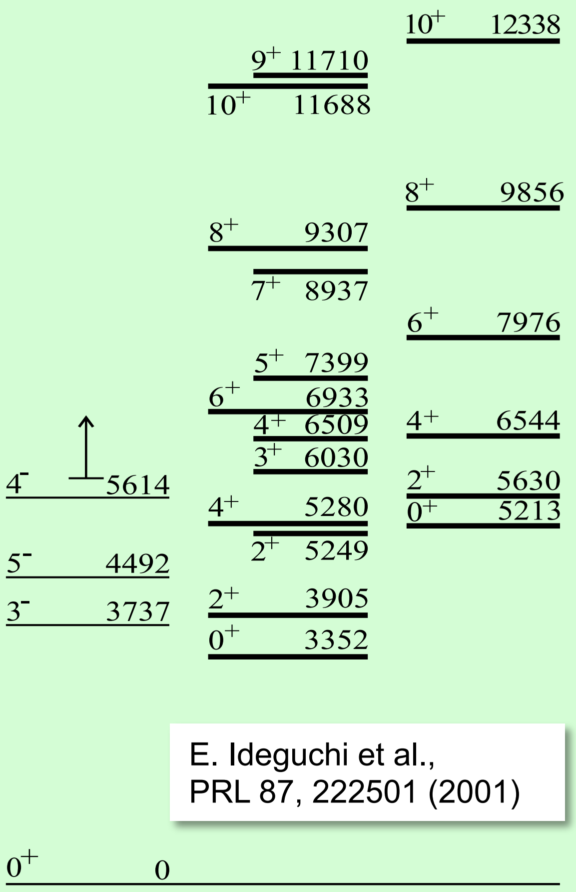
Shape coexistence appears to be universal, and it is essential to identify its occurrence at low energy.




R&W Figs. 1.73, 1.74

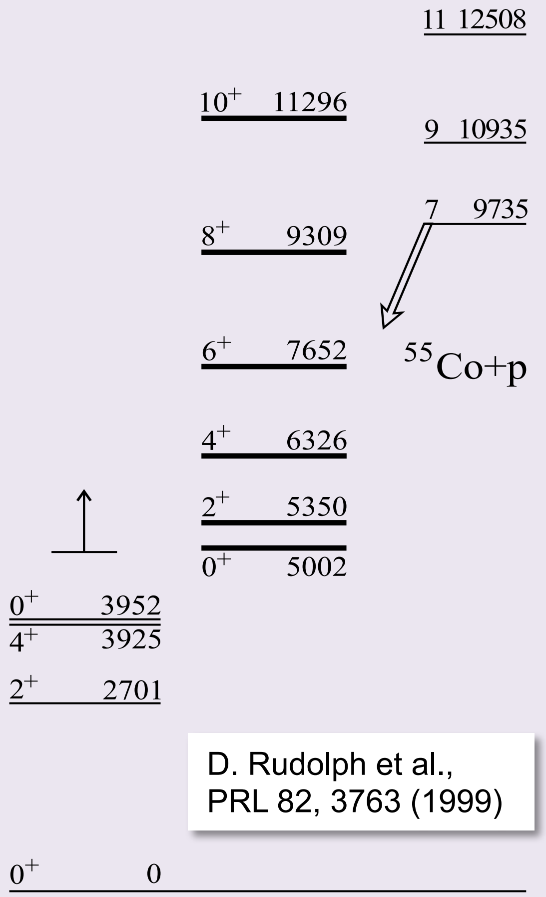
# Excited $0^+$ states at closed shells: Shape coexistence in the double-closed shell nuclei $^{40}Ca$ and $^{56}Ni$

Figure from K. Heyde and J.L. Wood, Rev. Mod. Phys. 83, 1467 (2011)




E. Ideguchi et al., PRL 87, 222501 (2001)

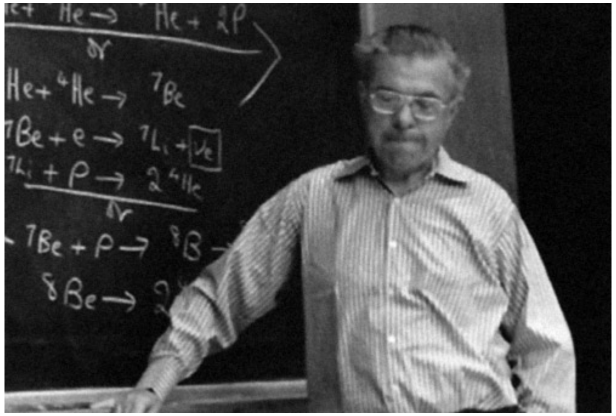
$^{40}$Ca




D. Rudolph et al., PRL 82, 3763 (1999)

$^{56}$Ni

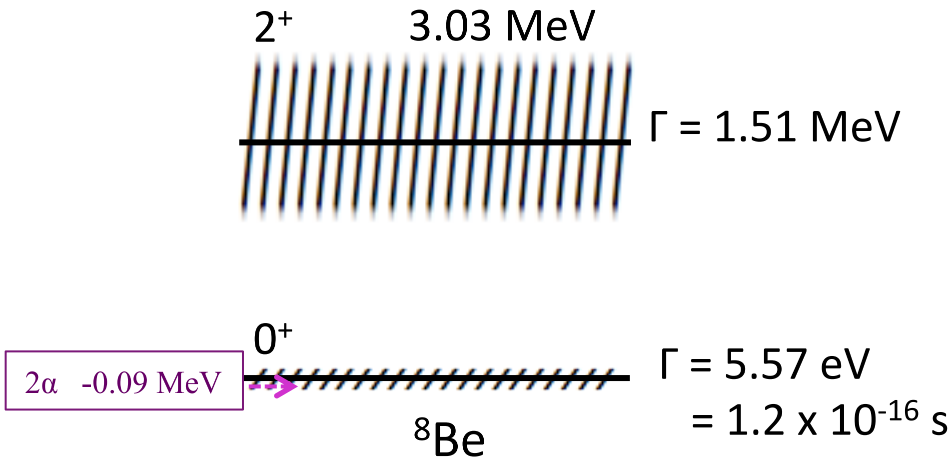
# The Hoyle state (7.65 MeV state in $^{12}C$)

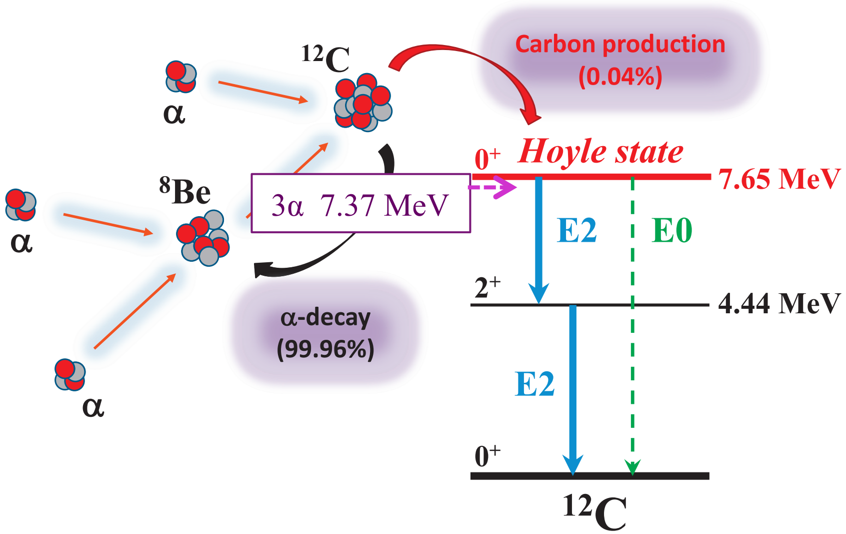


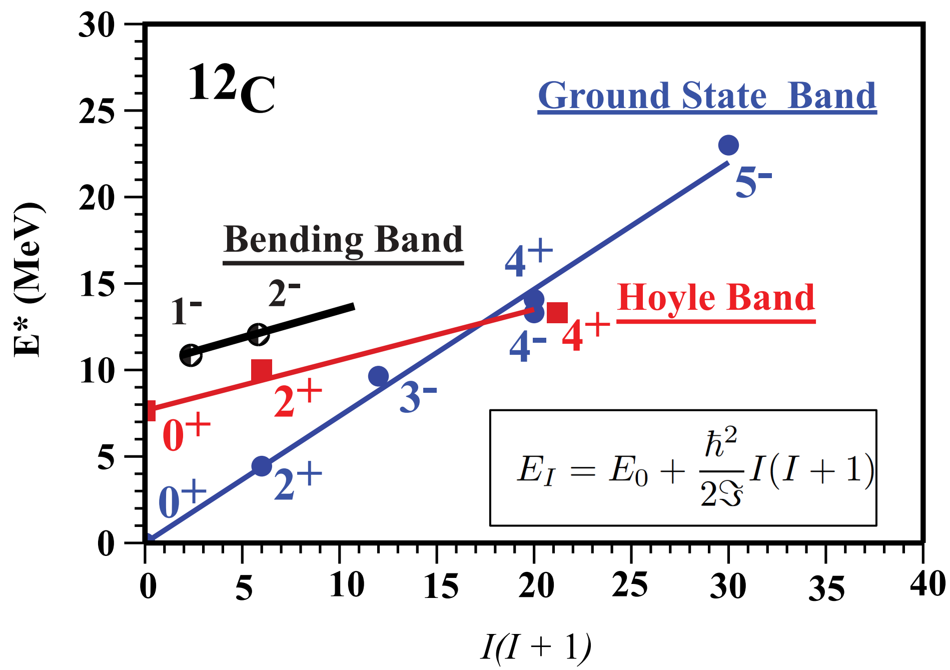
Sir Fred Hoyle (1915-2001)

Helium fusion in stars

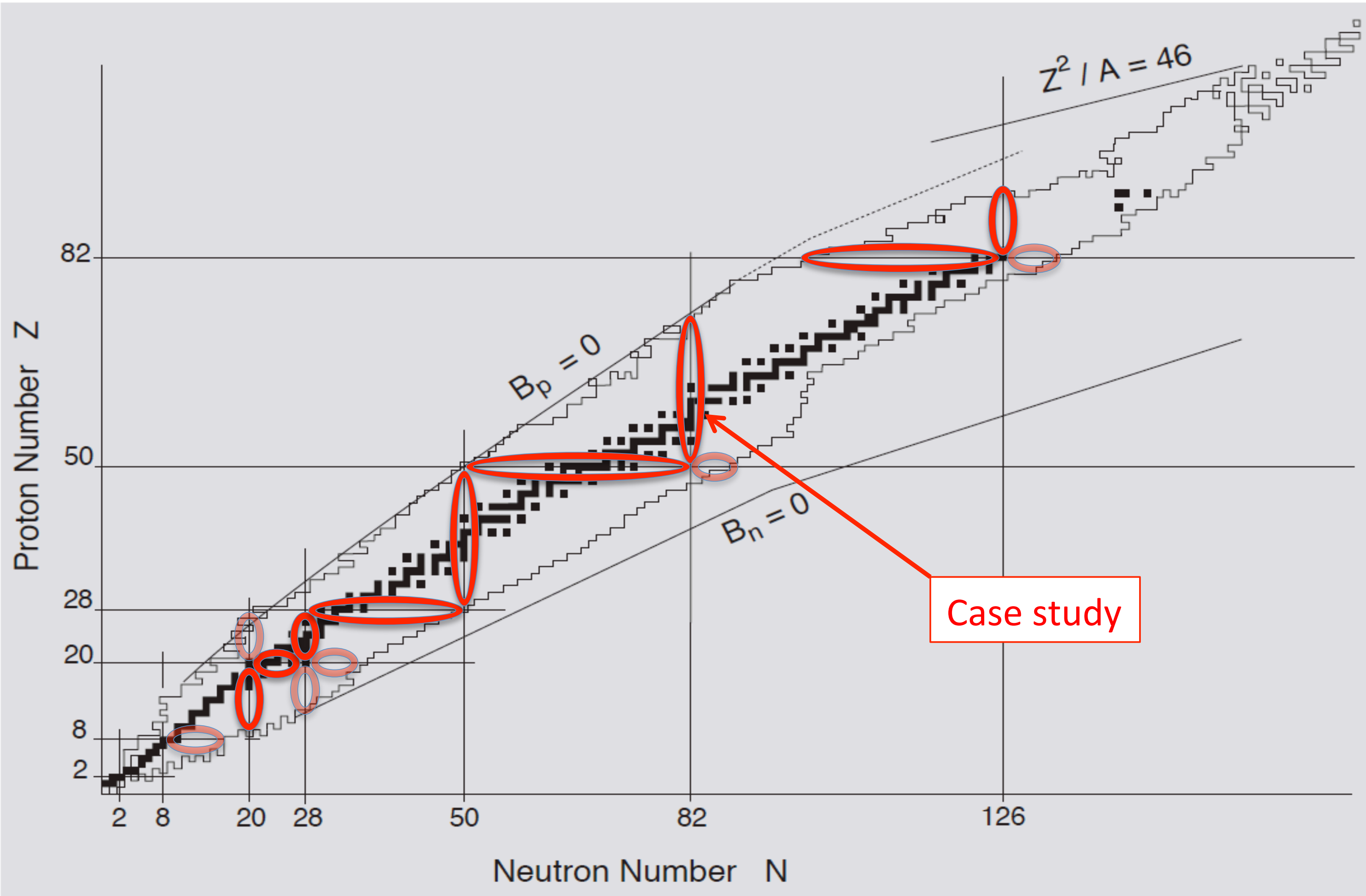
F. Hoyle, Astrophysical J. Suppl. Ser. **1** 121 1954



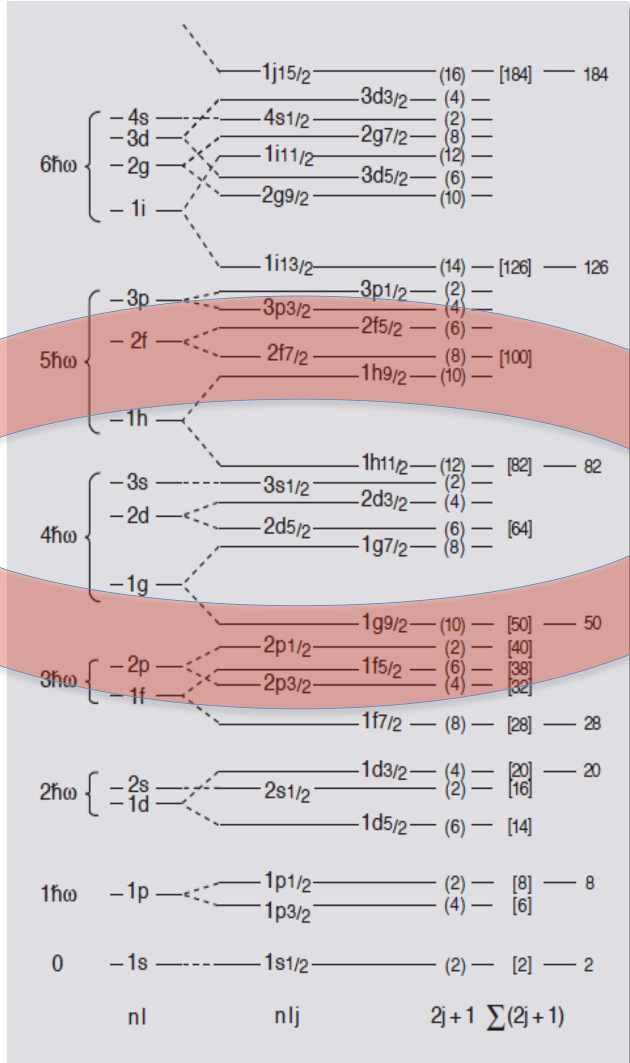



# Singly closed shell nuclei




R&W Fig. 1.2

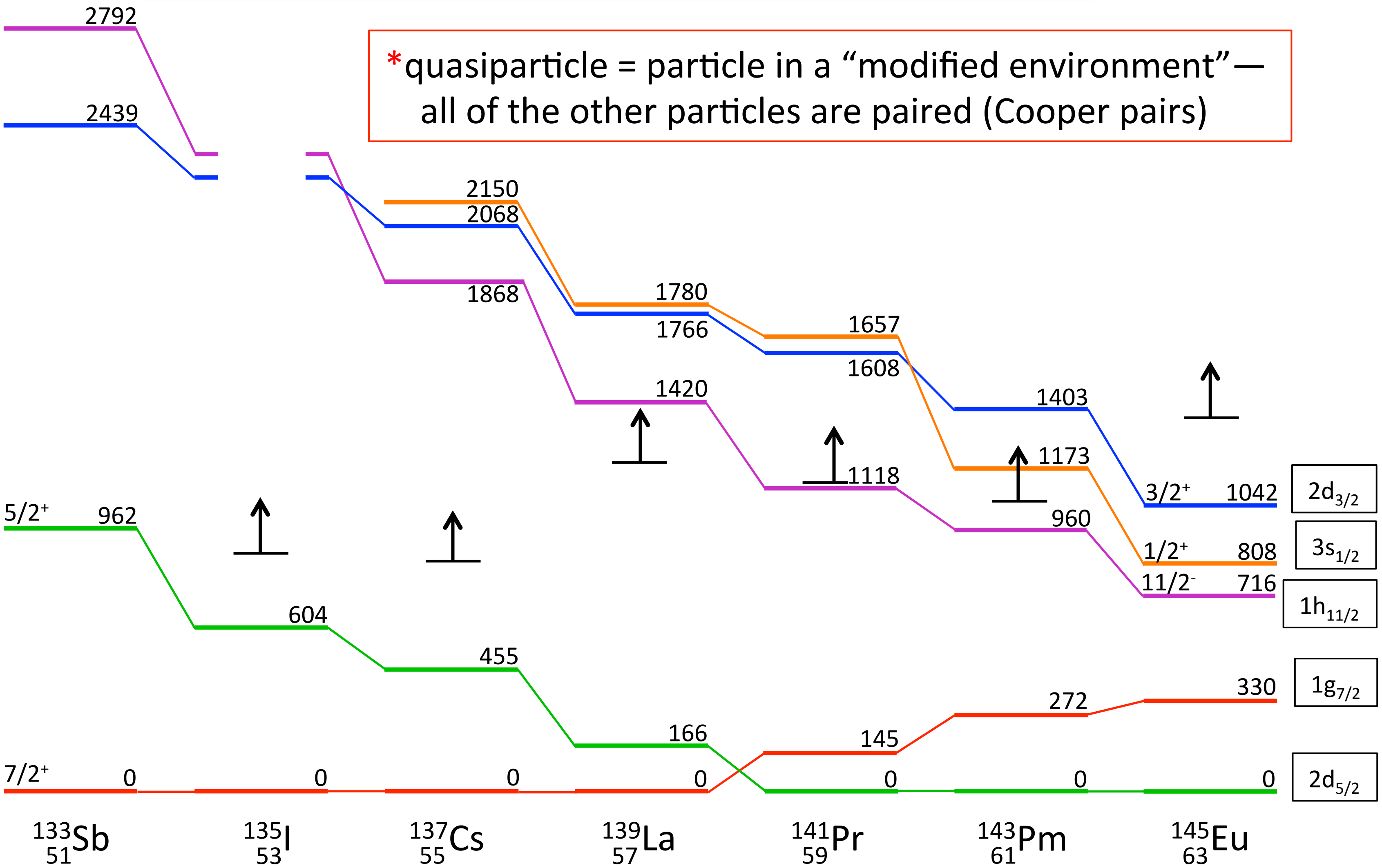
# Shell-model states:
# many-particle bookkeeping in spherical nuclei



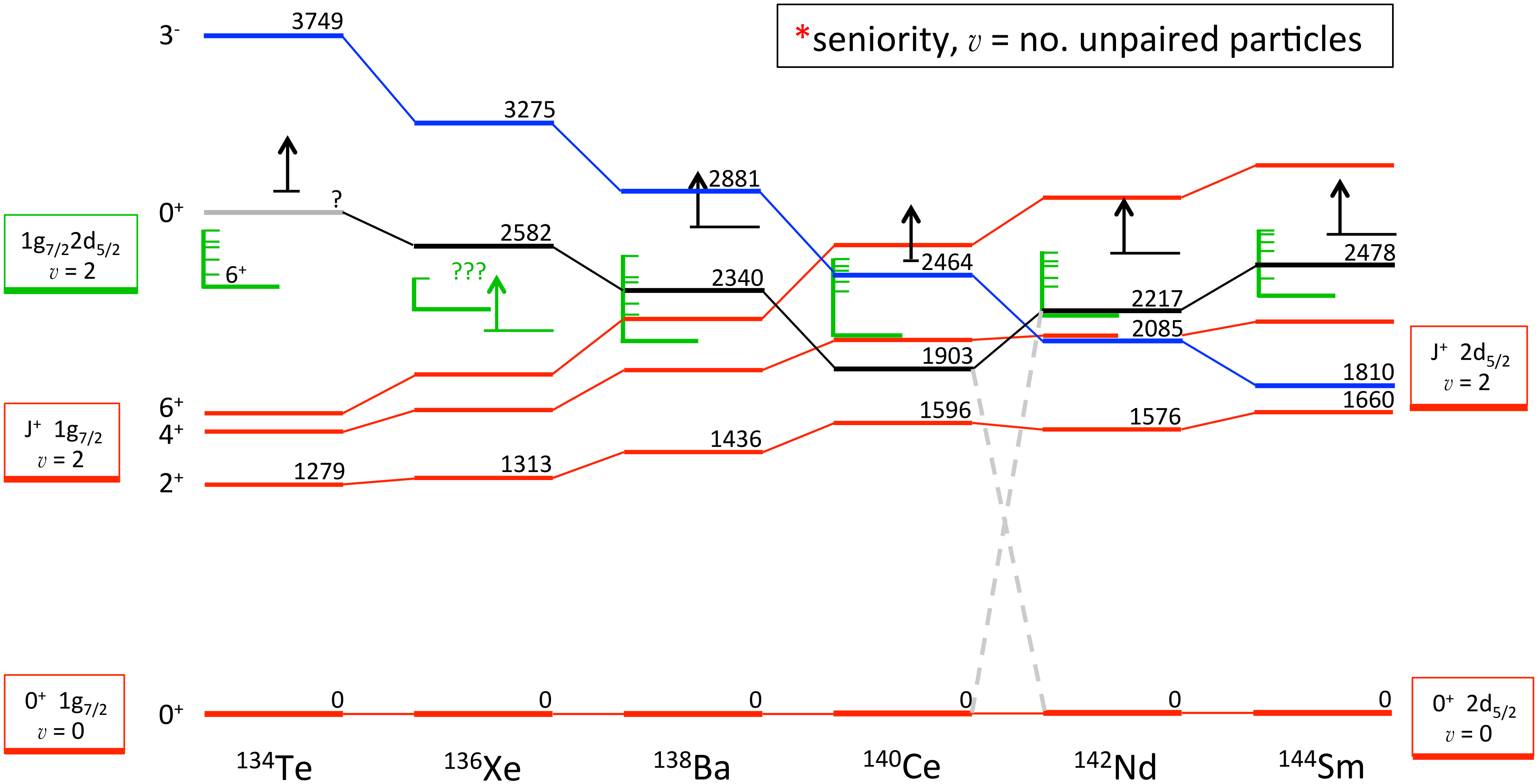

R&W Fig. 1.17

# N = 82: proton single-quasiparticle* states

*quasiparticle = particle in a "modified environment"—all of the other particles are paired (Cooper pairs)



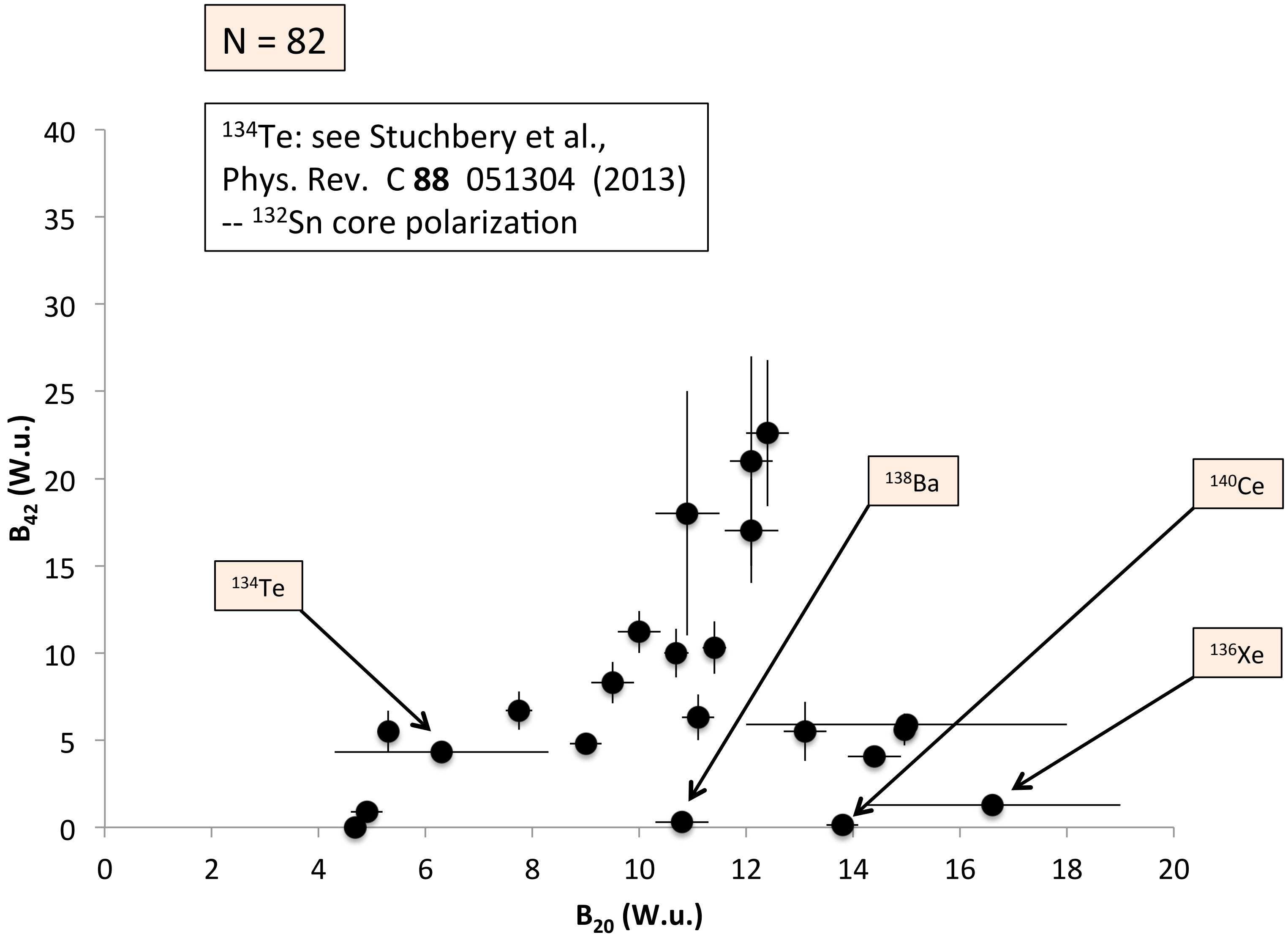
# N = 82 $g_{7/2} + d_{5/2}$-dominated seniority* structure




| $B_{20}$(W.u.) | $6.3^2$ | $16.6^{24}$ | $10.8^5$ | $13.8^3$ | $12.0^2$ | $11.9^4$ |
|---|---|---|---|---|---|---|

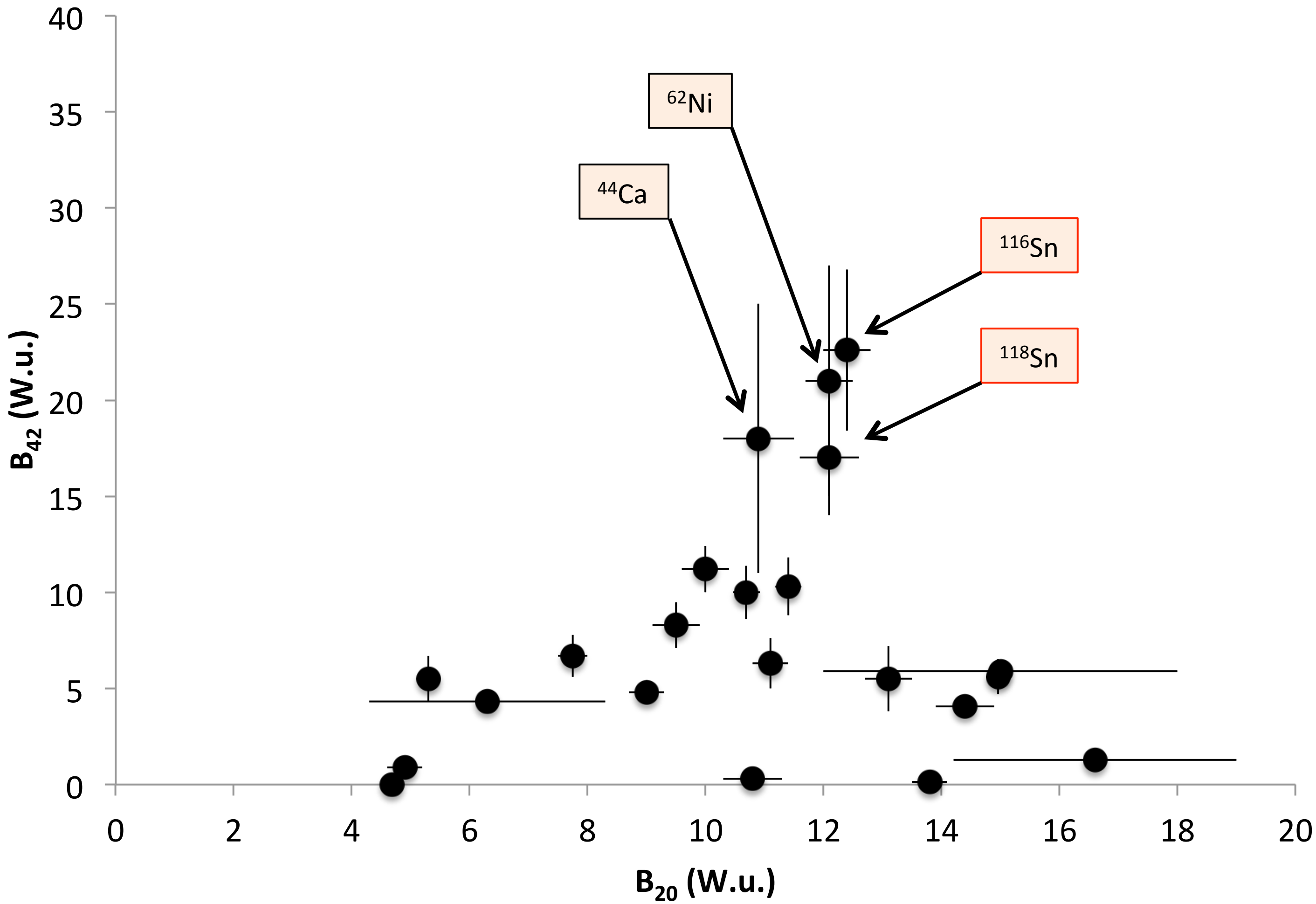
# m scheme

# $B_{42}$ vs. $B_{20}$ for singly closed-shell nuclei



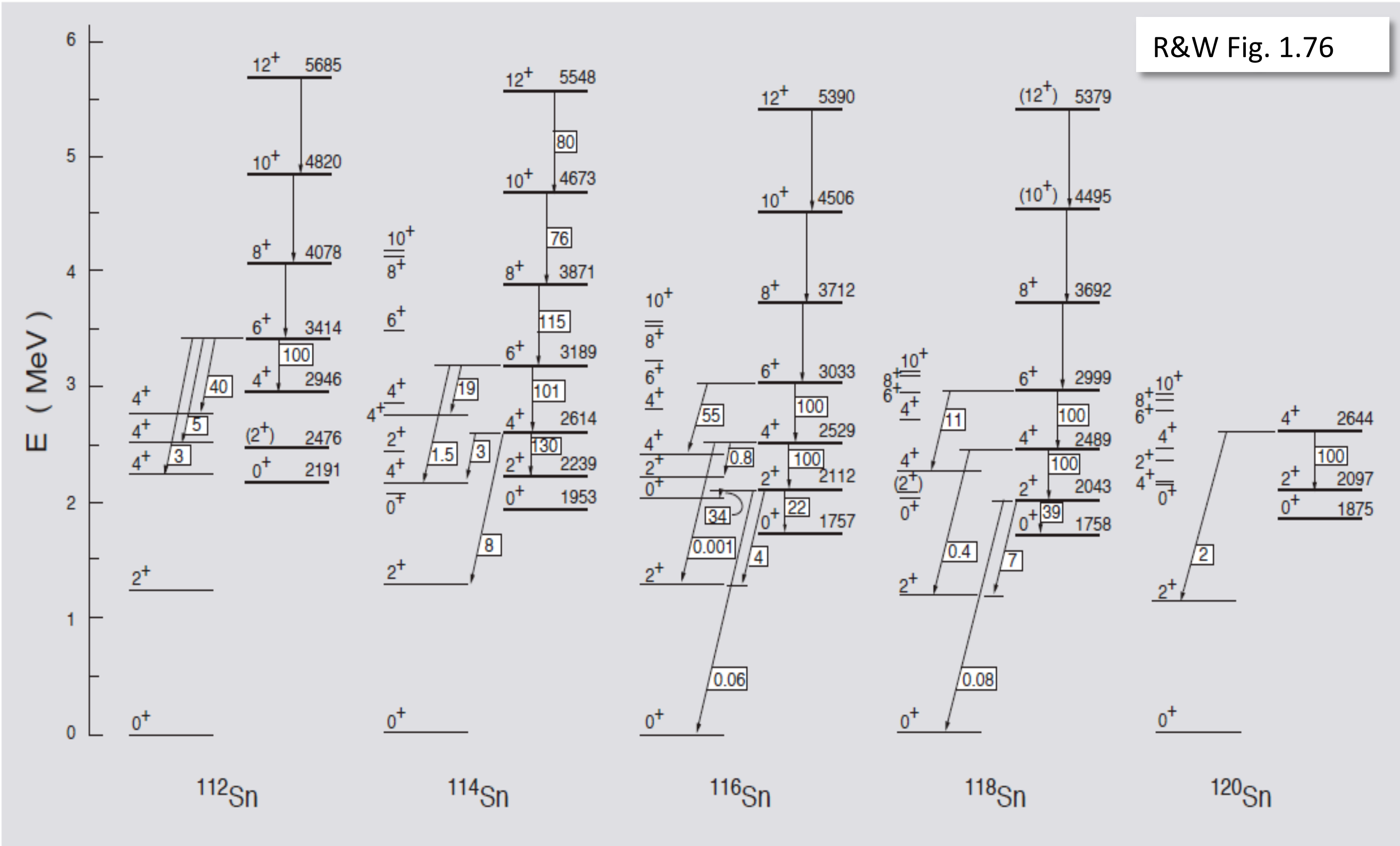
# $B_{42}$ vs. $B_{20}$ for singly closed-shell nuclei

enhancement due to mixing?



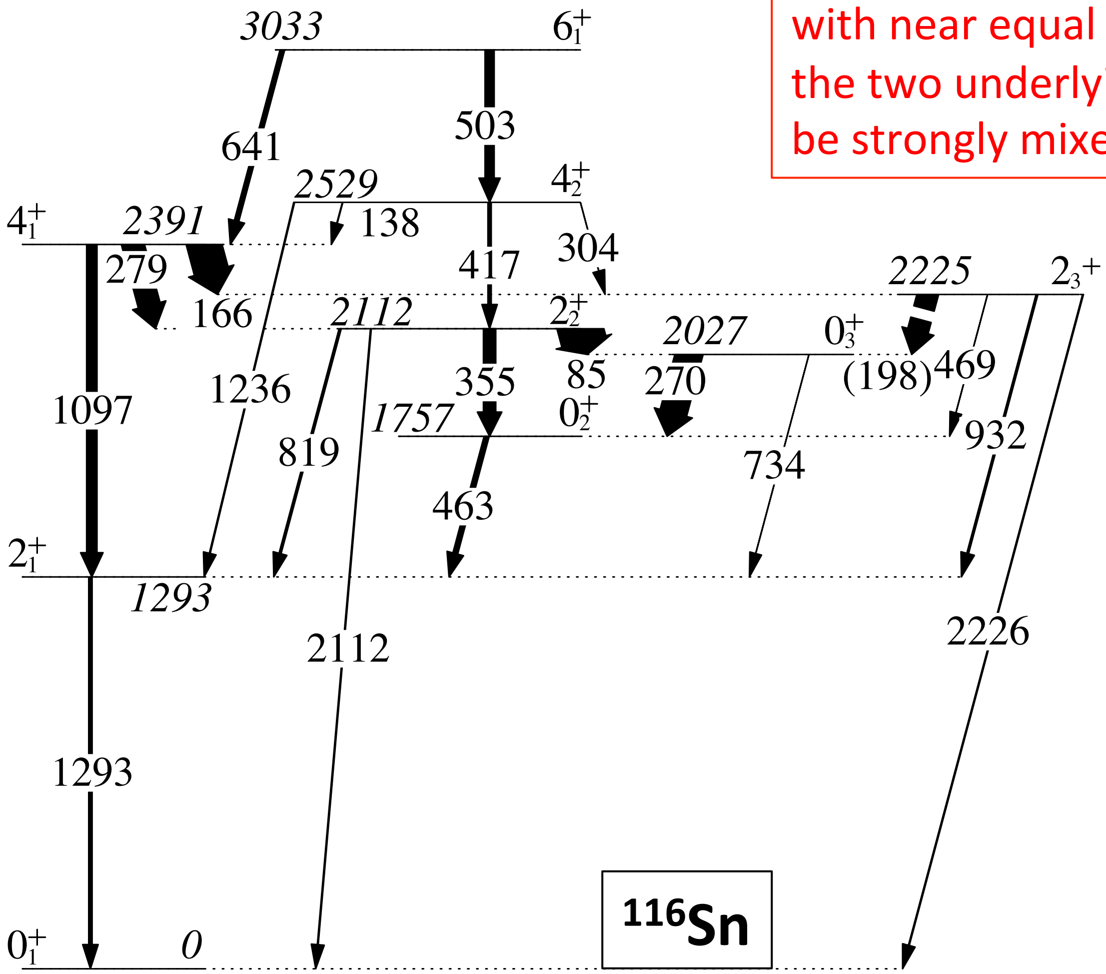
# Deformed bands built on excited $0^+$ states at closed shells: tin isotopes

B(E2)s in W.u. [100 = rel. value]

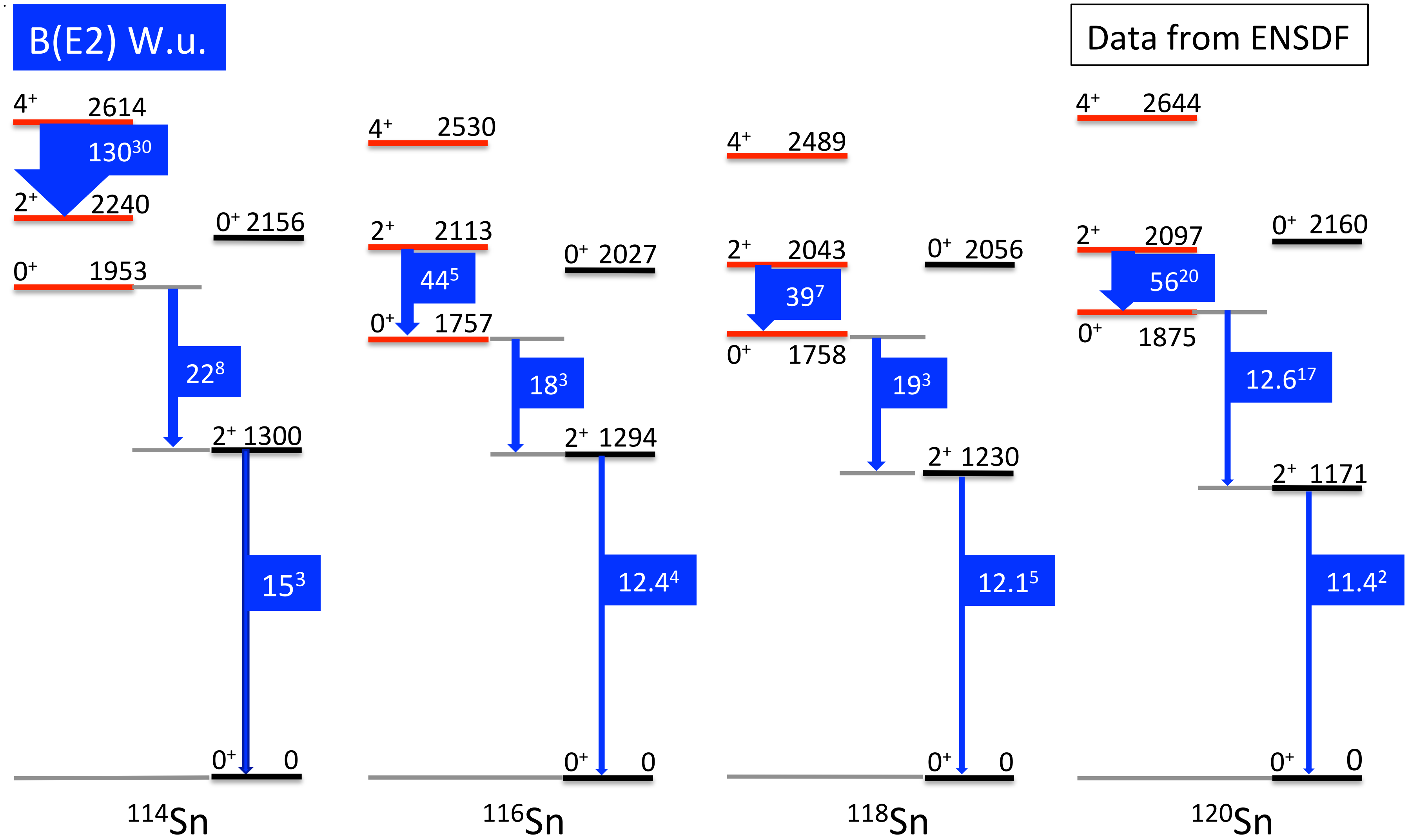


# Evidence for mixing of $4_1^+$ and $4_2^+$ configurations in $^{116}$Sn

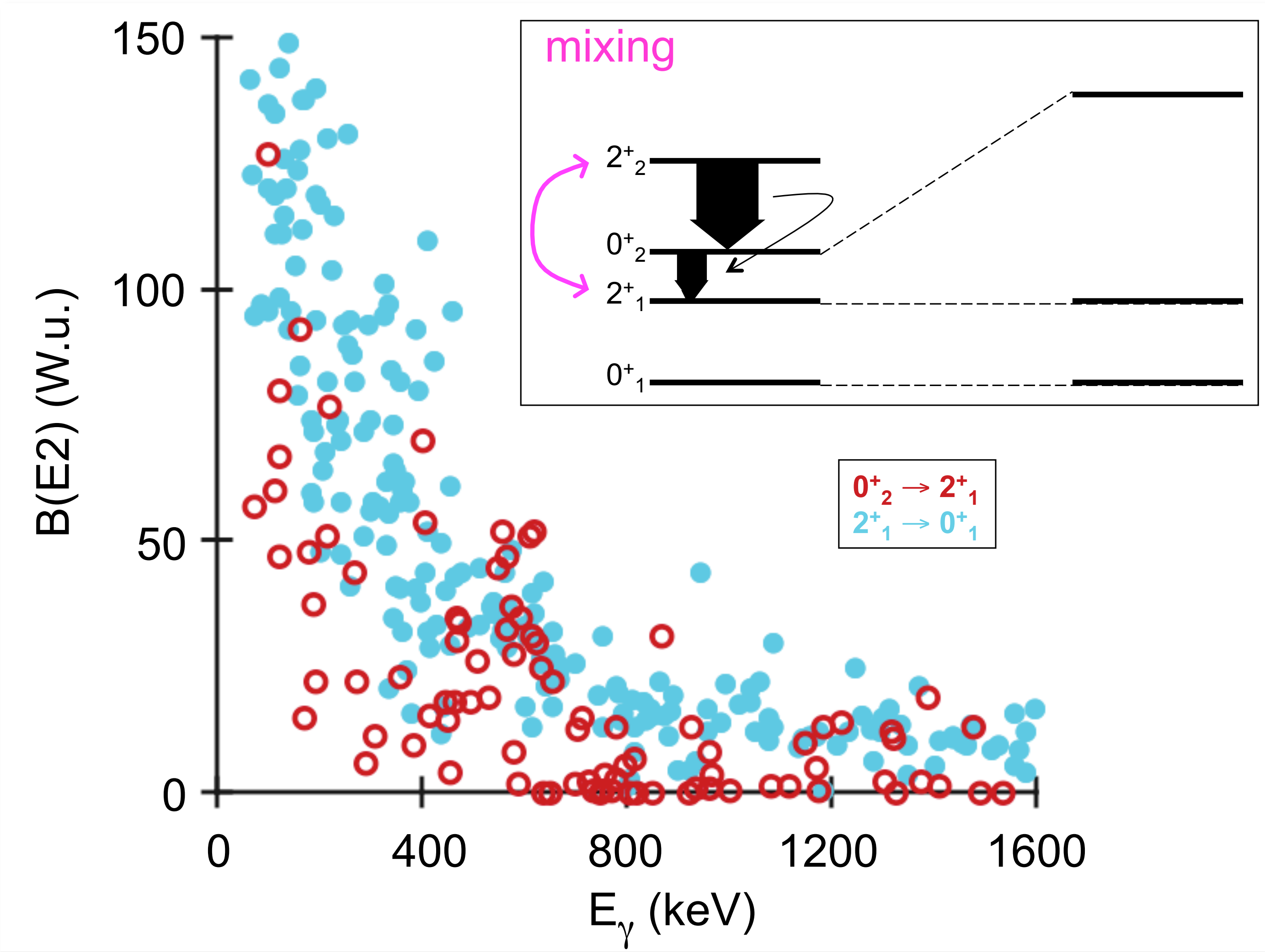
Decay of $6_1^+$ state to $4_1^+$ and $4_2^+$ states with near equal intensities indicates that the two underlying $4^+$ configurations must be strongly mixed.



# E2 transitions associated with shape coexistence in $^{114-120}Sn$



## B(E2; $0_2^+ \rightarrow 2_1^+$) vs. E($0_2^+$) – E($2_1^+$): shape coexistence and mixing yields B(E2; $0_2^+ \rightarrow 2_1^+$) ~ $\alpha^2 \beta^2 (\Delta Q)^2$

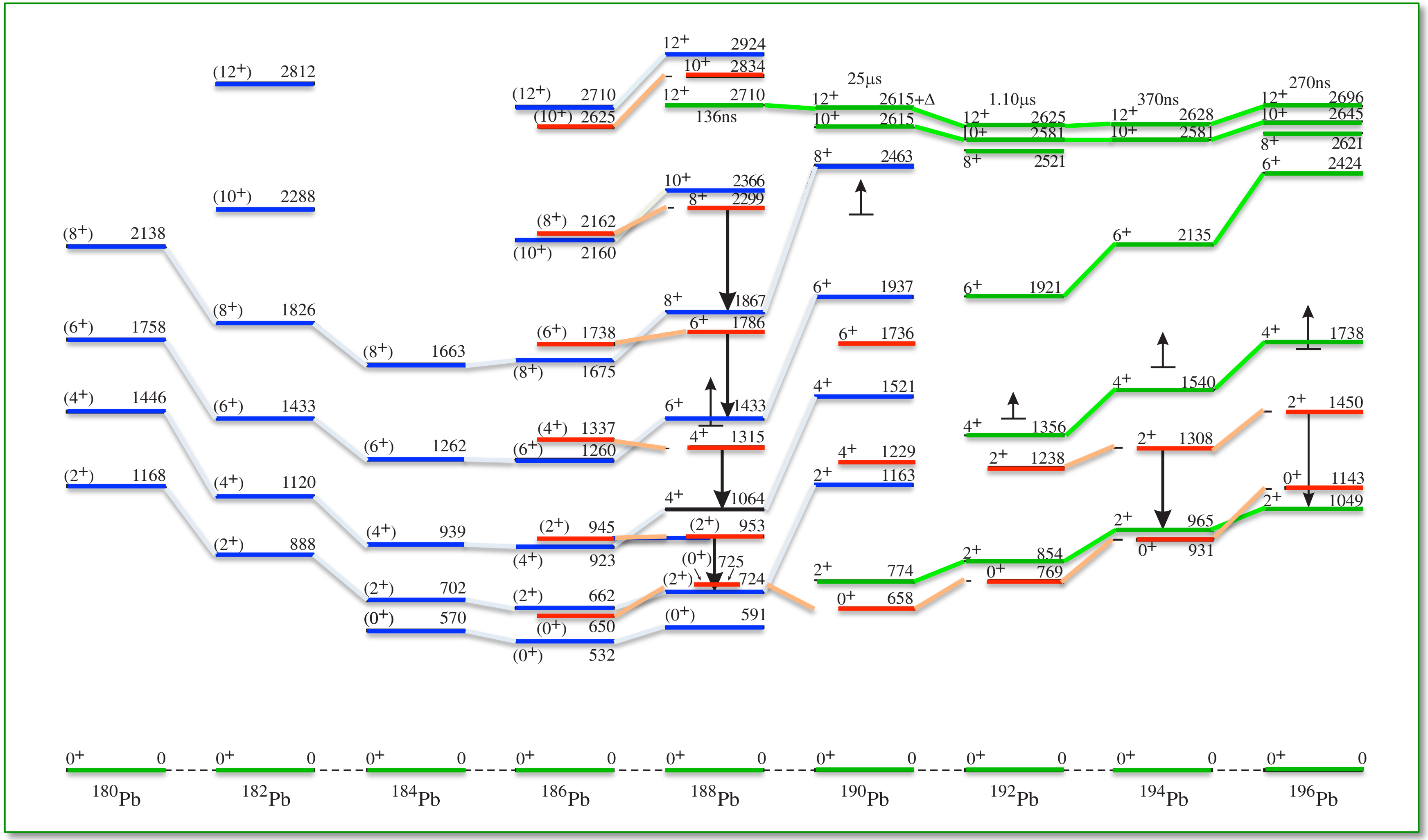



Recall:
$B_{02} = 5 \times B_{20}$

# Shape coexistence in the singly closed-shell lead (Z = 82) isotopes

Figure: Heyde & Wood

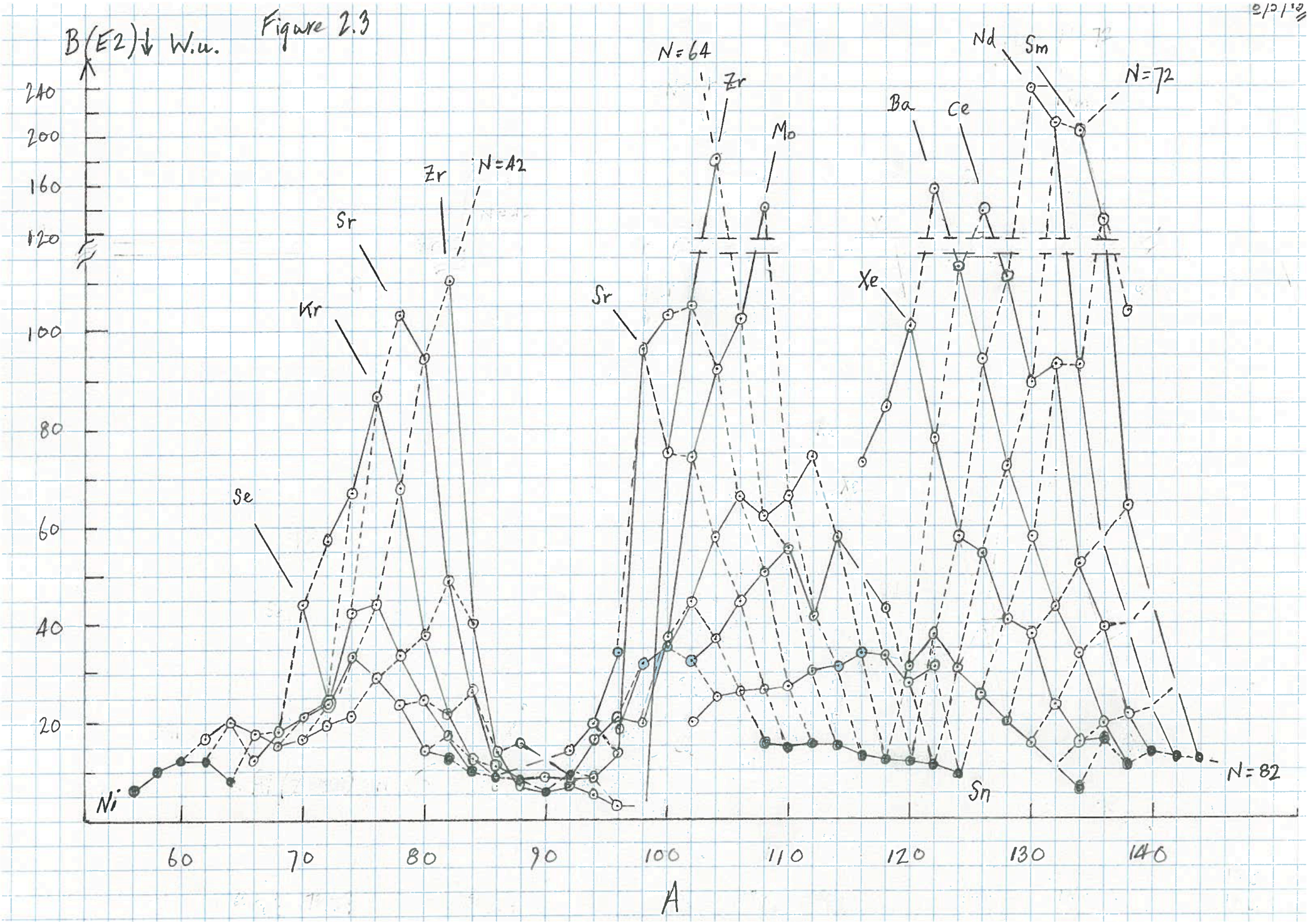
Heavy arrows indicate E0+M1+E2 transitions
$^{188}$Pb: G.D. Dracoulis et al., PR C **67** R 051301 2003



# LECTURE 2: DISCUSSION

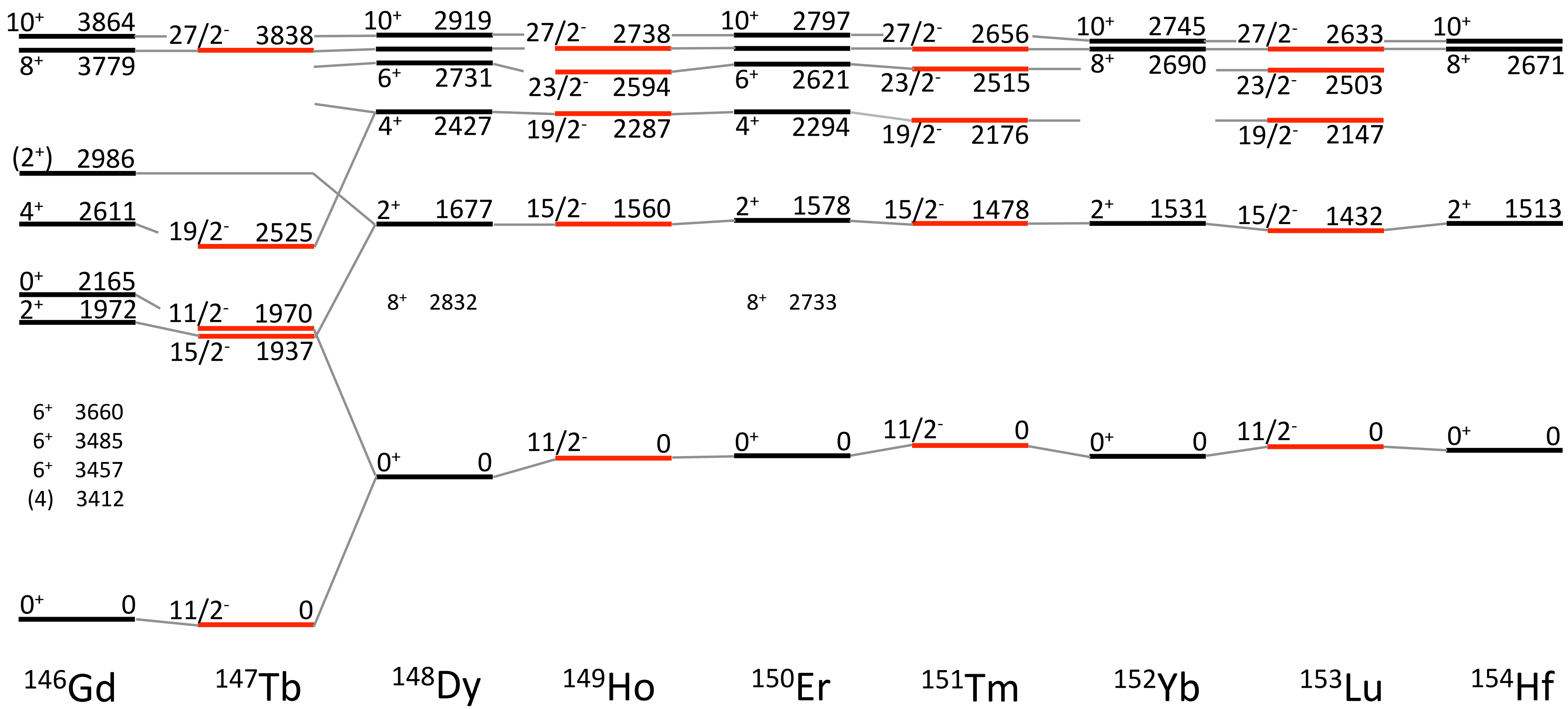
# Some questions

- If you plan a program of half-life measurements for $2_1^+$ states, which ones would you choose to re-measure in the $Z \geq 28$, $N \leq 82$ region?
- With respect to $^{208}Pb$, what did Heusler et al. achieve?



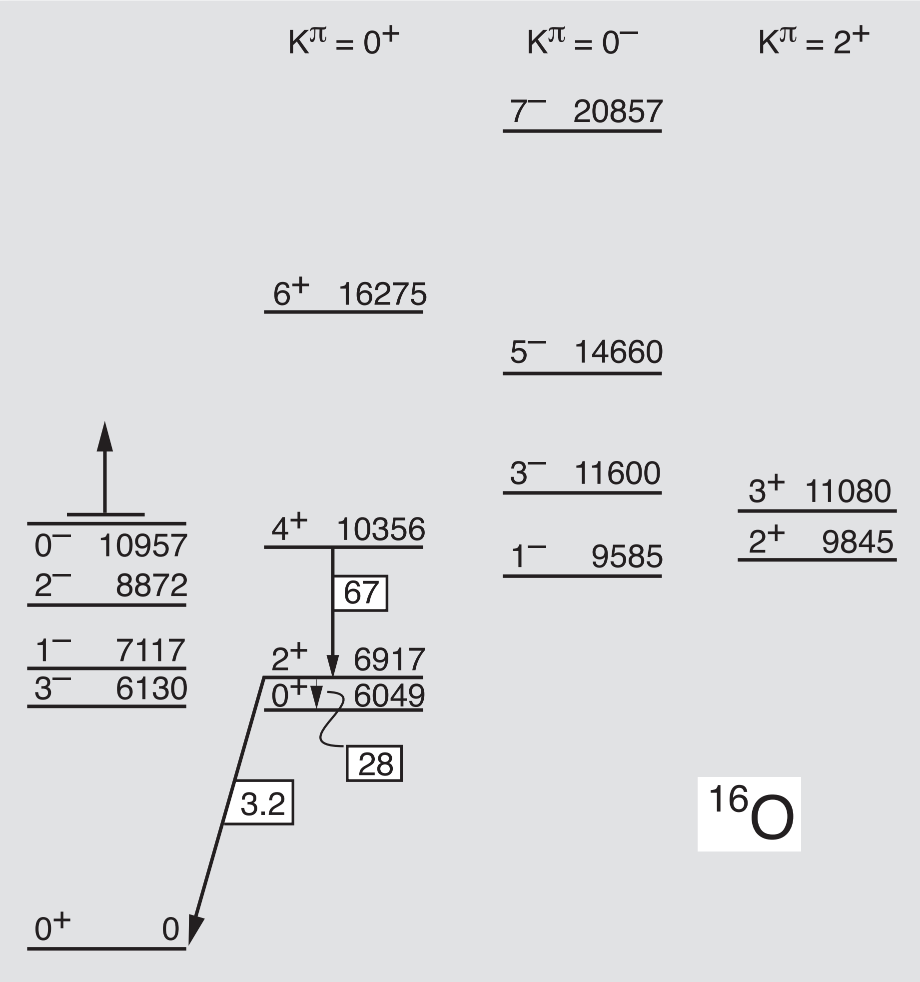

Values of B(E2; $2_1^+ \rightarrow 0_1^+$) in Weisskopf units (W.u.) for nuclei in the region Z ≥ 28, N ≤82. The heavy black dots mark the singly closed-shell nuclei at Z = 28, 50 and N = 50, 82. Solid lines connect isotopes and dashed lines connect isotones. Note the vertical compression above 100 W.u.

**Rowanwood Sect. 2.6 Fig. 2.6.3 v.7/24/16**




Seniority structures for the heavy N = 82 isotones. The structure of the higher-mass nuclei reflects the dominance of the $1h_{11/2}$ orbital. The structure of $^{146}$Gd exhibits a strong “depression” of the ground state energy as a result of the $(3s_{1/2})^2$, v = 0 configuration mixing with the $(1h_{11/2})^2$, v = 0 configuration. A similar ground state depression occurs in $^{147}$Tb for v = 1 configurations. The strength of the mixing of these configurations can be inferred to be ~ 1 MeV, by visual inspection. The states with $J^\pi = 4^+, 6^+$ in $^{152}$Yb and $^{154}$Hf are by-passed in the decay of the $8^+$ state by way of lower-lying $5^-$ and $7^-$ states. The $10^+$ state is known to influence the decays in $^{154}$Hf through the isomeric nature of the decay, but the very low energy of the $10^+ \rightarrow 8^+$ transition was outside of the range of sensitivity of the measurements made. There are candidate $6^+$ states known in $^{146}$Gd, but an unambiguous assignment has not been made. The $2d_{3/2}$ orbital also is influencing the low-energy structure of $^{146}$Gd. The energies are arbitrarily normalized at spin 8 and 27/2.

# Shape coexistence in the doubly closed-shell nucleus $^{16}O$




R&W Fig. 1.73

Energies of states are given in keV.

B(E2) values are given in W.u.

States on the far left are spherical.

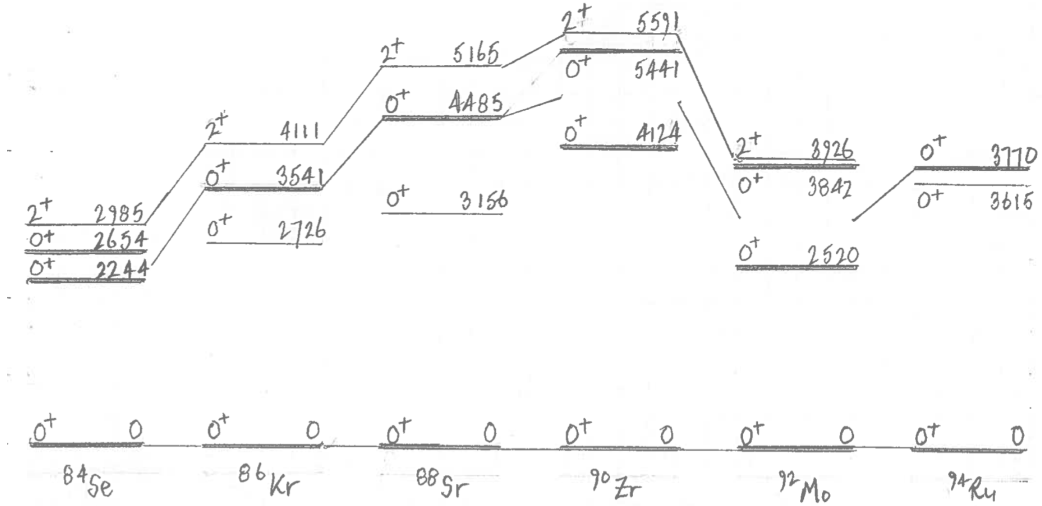
The beginnings of three deformed bands, with K = 0, 0, 2, are shown.

H. Morinaga, PR **101** 254 1956

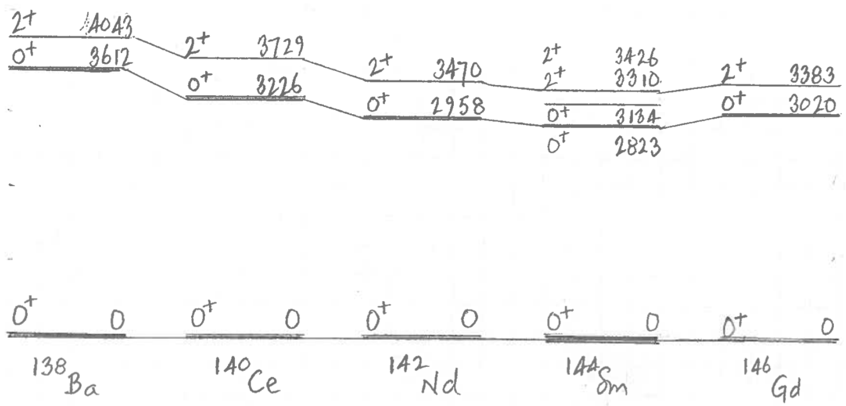
# Shape coexistence at closed shells: the N = 50, 82 isotones

$^{80}Ge^{*}$ $E(0_2^+) = 639$ keV, see:
A. Gottardo et al.,
PRL **116** 182501 (2016)
$^{*}N = 48$

N = 50



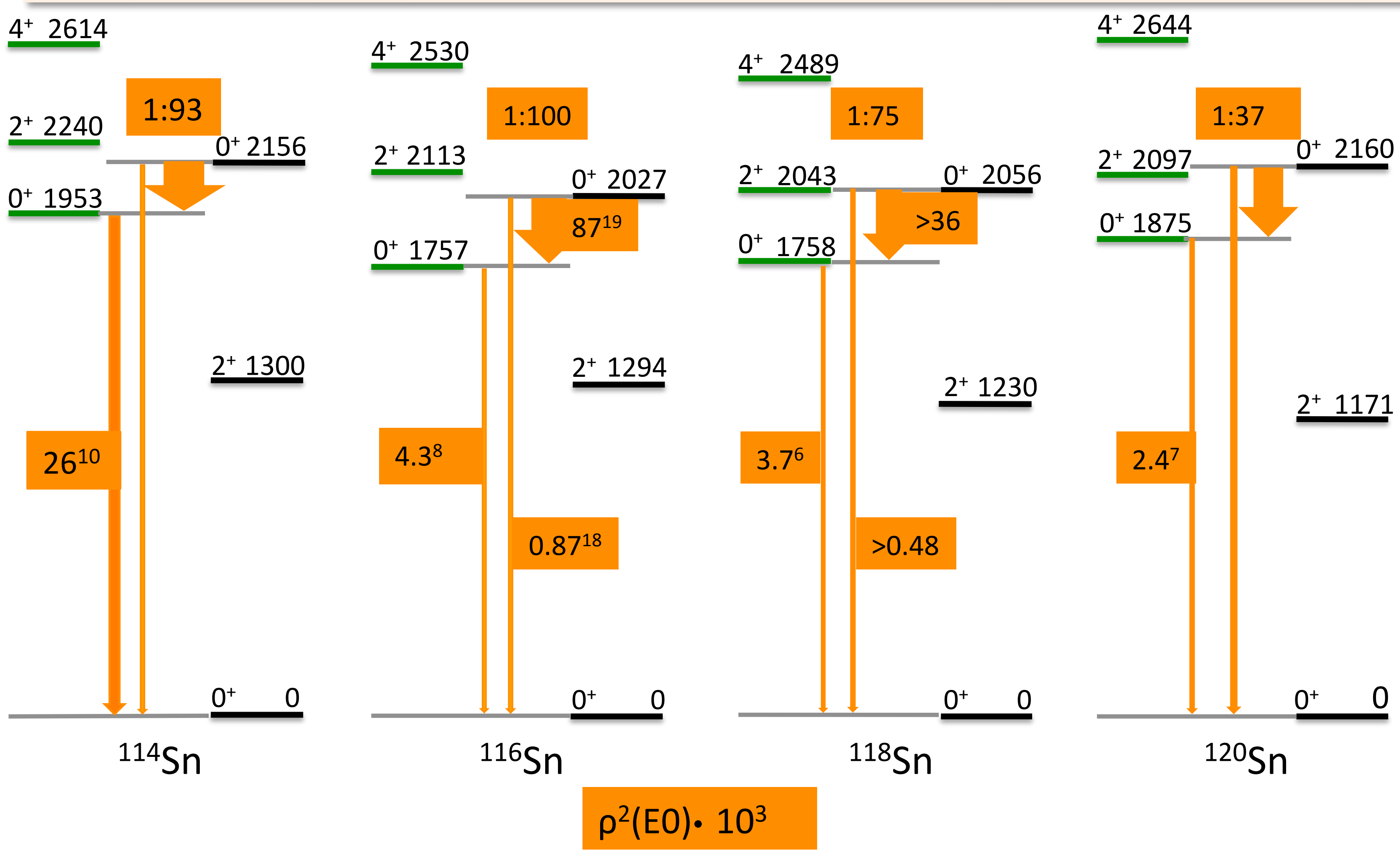

N = 82



# E0 transitions associated with shape coexistence in $^{114-120}Sn$

J. Kantele et al., ZP A289, 157 (1979)    T. Kibedi and R.H. Spear, ADNDT 89, 277 (2005)

Mixing of close lying configurations with different mean-square charge radii produces E0 strength



# E0 Transitions: shape coexistence and mixing

E0 transition strengths are a measure of the off-diagonal matrix elements of the mean-square charge radius operator.

Mixing of configurations with different mean-square charge radii produces E0 transition strength.

$$\rho^2(E0) = \frac{1}{\Omega\, \tau(E0)}$$

"Electronic factor"

$$\Omega = \Omega(Z, \Delta E) = \Omega_K + \Omega_{L_1} + \cdots + \Omega_{e^+e^-}$$

Monopole strength parameter

$$\rho_{if}(E0) = \frac{\langle f | \sum_j e_j r_j^2 | i \rangle}{eR^2} \equiv \frac{\langle f | m(E0) | i \rangle}{eR^2} \equiv \frac{M_{if}(E0)}{eR^2}$$

$$|i\rangle = \alpha|1\rangle + \beta|2\rangle \,, \quad |f\rangle = -\beta|1\rangle + \alpha|2\rangle$$

$$M_{if}(E0) = \alpha\beta\left\{\langle 2|m(E0)|2\rangle - \langle 1|m(E0)|1\rangle\right\} + (\alpha^2-\beta^2)\langle 1|m(E0)|2\rangle$$

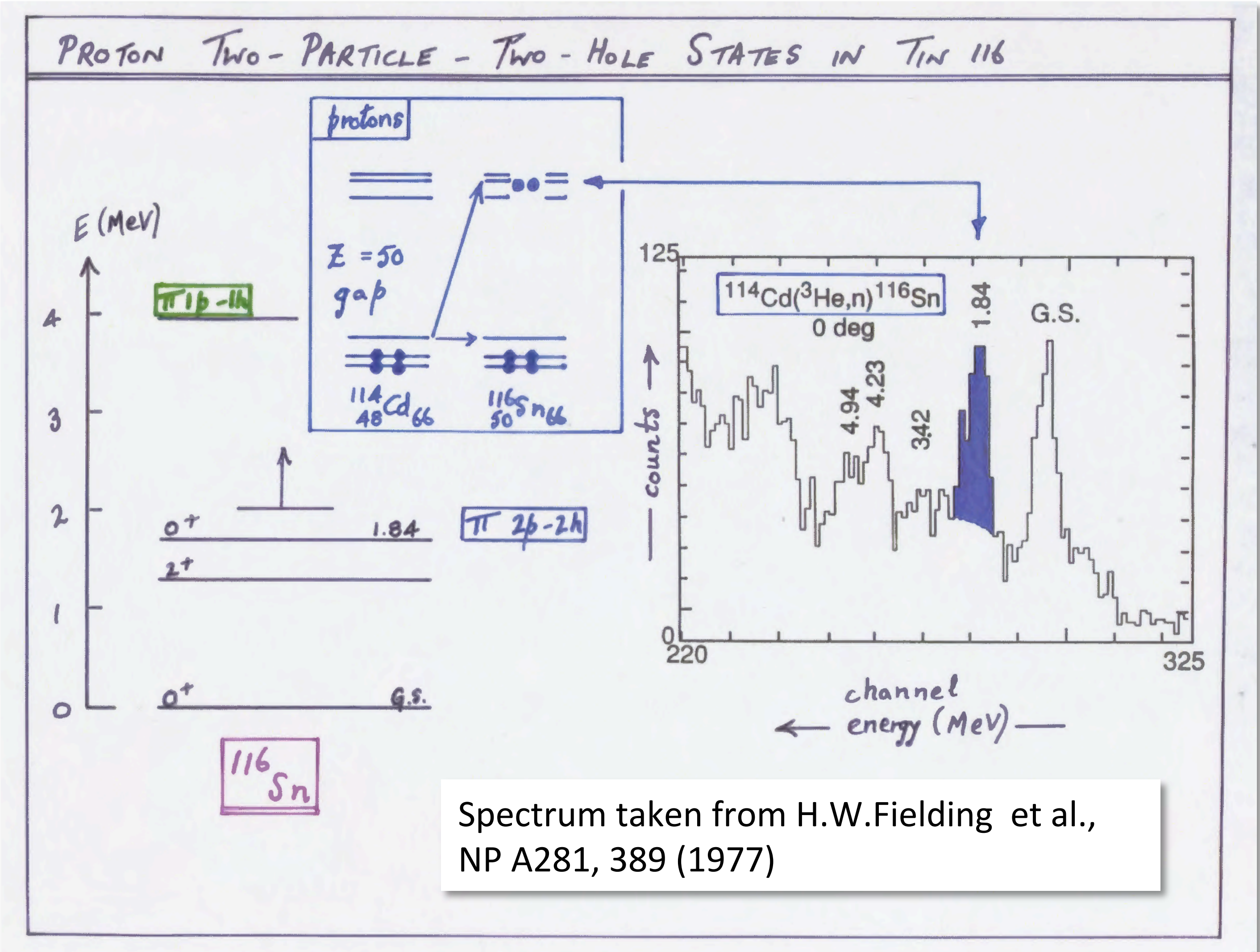
$$\boxed{M_{if}(E0) \simeq \alpha\beta\, \Delta\langle r^2\rangle}$$

Ω values: http://bricc.anu.edu.au

τ: partial lifetime for E0 decay branch

J. Kantele et al. Z. Phys. A289 157 1979
and see
JLW et al. Nucl. Phys. A651 323 1999

# The nature of the shape coexisting state in $^{116}$Sn revealed by ($^3$He,n) transfer reaction spectroscopy




Spectrum taken from H.W.Fielding et al., NP A281, 389 (1977)

# Two-state mixing